What Fits

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Overview of Lesson

This activity has students explore what a line of best fit is and where a line of best fit should be placed on a scatterplot of data with a linear association. This is done through students' completion of an activity investigating the relationship between the drop height of a golf ball and the height of its first bounce, as well as analysis of other students' work on the same activity. Students will learn what the line of best fit means and appropriate criteria for its placement.

GAISE Components

This activity follows all of the four components of statistical problem solving put forth in the Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report. This is a Level B activity.

Common Core State Standards for Mathematical Practice

3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
7. Look for and make use of structure.

Common Core State Standards Grade Level Content (Grade 8)

8. SP. 1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
8. SP. 2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
8. SP. 3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.

NCTM Principles and Standards for School Mathematics
Data Analysis and Probability Standards for Grades 6-8

Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them:

- formulate questions, design studies, and collect data about a characteristic shared by two populations or different characteristics within one population;
- select, create, and use appropriate graphical representations of data, including histograms, box plots, and scatterplots.

Select and use appropriate statistical methods to analyze data:

- discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stem-and-leaf plots, box plots, and scatterplots.

Develop and evaluate inferences and predictions that are based on data:

- make conjectures about possible relationships between two characteristics of a sample on the basis of scatterplots of the data and approximate lines of fit;
- use conjectures to formulate new questions and plan new studies to answer them.

Prerequisites

Prior to completing this activity, students should:

- be familiar with displaying bivariate quantitative data in scatterplots
- understand a linear function has a constant rate of change
- be able to find the *y*-coordinate of a point on a line given the line's equation and the point's *x*-coordinate
- be able to find the *y*-coordinate of a point on a line given the line's graph and the point's *x*-coordinate
- determine a line's slope and *y*-intercept given its graph and/or equation.

Learning Targets

Students will be able to:

- interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities
- describe patterns in scatterplots such as clustering, positive or negative association, and linear association
- explain why straight lines are used to model relationships between two quantitative variables with a linear association
- informally fit a straight line
- informally assess the model's fit by judging the closeness of the data points to the line
- analyze criteria for lines of best fit
- explain what the line of best fit is
- use the equation of a line of best fit to make a prediction and judge the reasonableness of the prediction
- interpret the slope and *y*-intercept of the equation of a line of best fit in context and judge the reasonableness of those interpretations.

Time Required

The time required for this activity is roughly 90 minutes.

Materials Required

For this activity, the students will need copies of the activity sheets, short pieces of stiff wire (available at hobby or craft stores) or spaghetti, and pencils.

Instructional Lesson Plan

The GAISE Statistical Problem-Solving Procedure

I. Formulate Question(s)

This activity will be driven by the teacher-directed question: **"What is the relationship between the drop height of a golf ball and the height of its first bounce?"** Conduct a discussion using the following questions:

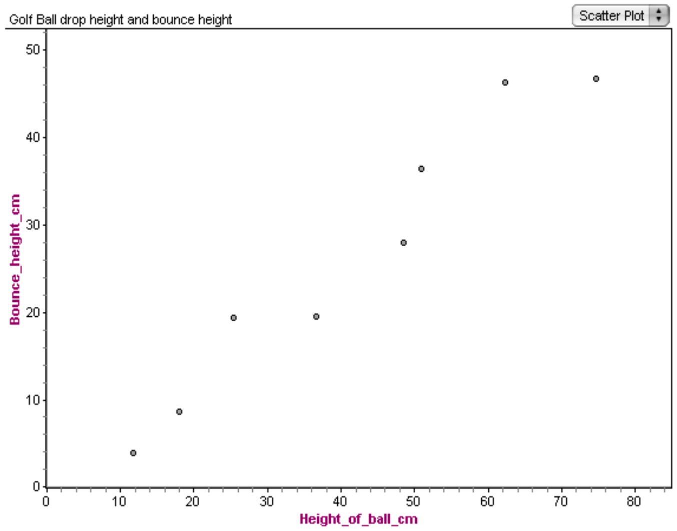
- What is our population of interest?
- What are the two variables of interest?
- Which variable is the explanatory variable? Why?
- Do you expect there to be a strong association between these two variables? Why?
- What form do you expect the relationship to follow (i.e., what function family would be used to model this relationship)? Why?
- What direction do you expect the association to take? Why?
- Do you expect a golf ball to bounce back up to its drop height? Lower than its drop height? Higher than its drop height?

An extensive discussion should take place surrounding these questions. A description of correct responses to the above questions is located in the answer key on page 14.

II. Design and Implement a Plan to Collect the Data

Option 1: Use pre-collected data.
Note: This pre-collected data is the data used in part two of the activity.
Present the students with the first page of the activity (page 9). Have them read the following introduction and scatterplot.

An eighth grade class was investigating how high a golf ball bounces when it is dropped from different heights. Students were given eight set heights to drop the ball from. Then they dropped a golf ball from each of those heights and measured how high the ball bounced back up. Below is the scatterplot they made of their data.



Option 2: Collect data.
Pair the students in the class. Distribute a meter stick to each pair and assign each pair a drop height (suggested drop heights are 10 cm, 15 cm, 20 cm, etc.). One golf ball will be used by the whole class to provide consistency. Each pair will drop the golf ball from their given height and measure how high it bounces on its first bounce. It is recommended that the meter stick be placed against a wall to make it easier to see the bounce height. Another option is to record a movie of the ball bounce with the meter stick behind it which can then be replayed at a slower speed to determine the bounce height more accurately.

The data from all of the pairs will be collected in a table. Finally, the students should make a scatterplot of the data on a piece of paper.

III. Analyze the Data

This phase will involve students in analyzing the scatterplot of bounce height vs. drop height for a golf ball from Phase II, placing a line of best fit on the graph, and using the line to predict how high the golf ball would bounce if dropped from 60 cm.

First, have the students interpret the scatterplot, comparing the actual plot to one(s) anticipated in Phase I. Also ask the students to describe any patterns in the scatterplot such as clustering, positive or negative association, and linear association.

Once students have determined that a linear model is most appropriate, have the students individually do as directed in Part One of the activity:

> Determine where you think the line of best fit should be placed on the scatterplot by manipulating a piece of spaghetti or wire. Draw your line on the scatterplot once you have determined its best placement.

Students should be encouraged to play around with the location of the line of best fit by moving their piece of spaghetti or wire around the scatterplot until they feel its location best fits the data. Then the line should be drawn on the scatterplot.

Finally, each student should use his or her line to predict the height the golf ball will bounce to if it is dropped from 60 centimeters then evaluate whether that prediction is reasonable. It is expected that students will determine their predicted value graphically by reading up from the given drop height of 60 cm on the *x*-axis to the line, then over to the *y*-axis to determine the predicted bounce height. The reasonableness of the prediction should be assessed by comparing the predicted value to bounce heights from drop heights close to 60 cm.

IV. Interpret the Results

This phase of the lesson is divided into four parts.

(1) Lead a class discussion of the line of best fit as determined by the students individually. Include in your discussion:

- What things did students consider when determining the line's placement?
- What were the student-created criteria for determining where to place the line?
- Does the line match the relationship students anticipated in Phase I: Formulate a question?
- What are some reasons why the data are not perfectly linear?

(2) This part of the lesson focuses on Part Two of the activity, where students will examine other students' work in order to critique the types of reasoning performed by other students while placing the line of best fit.

Distribute the remaining pages of the activity (pages 10-13). Help students understand what they are asked to do in Part Two of the activity, as described at the top of page 10:

In the class that collected the data, the teacher gave the students a copy of the scatterplot and asked each of them to find the line of best fit for the data. The teacher then asked each student to explain why they put the line where they did, defining the criterion for the line of best fit.

Some sample student responses to this task are on pages 10-13. For each student's response,

1) Describe what the student's line of best fit tells them about the relationship between the bounce height of a ball and the height it was dropped from.
2) Interpret the slope and *y*-intercept of the student's line in the context of this data set. Do their interpretations seem reasonable in this context?
3) Use the student's line to predict the height the golf ball will bounce to if it is dropped from 60 centimeters. Is the prediction reasonable?
4) Analyze the student's criterion. Will the criterion always work to produce a line which accurately models any data set? If it will, explain why. If it won't, draw at least one example of a scatterplot with the line placed using that criterion and explain why the criterion produces a bad line of best fit.

Give students time to complete Part Two in groups.

At this point in the lesson, a teacher may help students begin to develop a way to distinguish a "good" trend line from a "bad" one. For example, looking at the sum of the absolute values of the residuals (residual = observed *y*-value minus predicted *y*-value for a given *x*-value) would be one way to begin to do that.

(3) Have students revisit their work in Part One and make corrections if necessary.

(4) Have students answer two summary questions, listed at the end of the activity sheet:

1) What is the best criterion to use when determining where the line of best fit is for any data set? Justify your response.
2) Another student asks you “What *is* the line of best fit?”. Write your response to this student.

Possible correct responses include:
Question 1: I think the best criterion is the one that makes the total distances of the points above and below the line the same. That way it gets the line in the middle and follows the trend of the data set.
Question 2: It’s a line that shows your best guess as to where the data would be for those values that you don’t have.

Assessment Questions

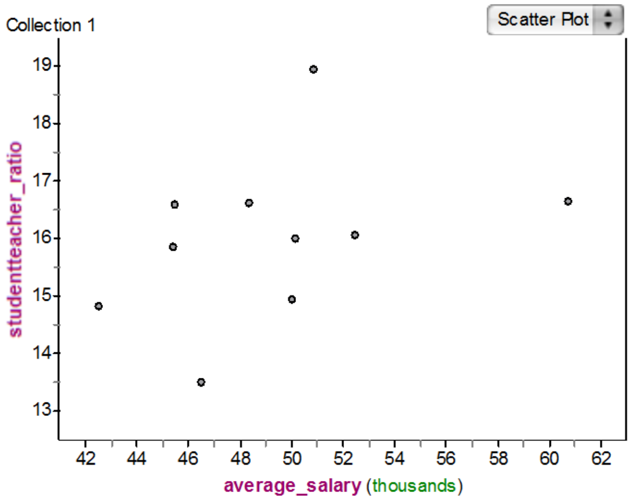
The table below gives the average salary for full-time teachers and the student/teacher ratio (number of students per teacher) for 10 public school districts in Pennsylvania in 2005.

district#	Average salary for full-time teachers	Student/ teacher ratio
1	$60,697	16.64
2	$50,156	15.99
3	$45,448	16.56
4	$46,504	13.49
5	$52,424	16.05
6	$42,508	14.81
7	$50,831	18.94
8	$48,376	16.61
9	$49,992	14.94
10	$45,368	15.85

1) Make a scatterplot of the data with average salary as the explanatory variable.
2) Explain reasons why there is "scatter" in the plot.
3) Describe trends in the plot such as clustering, positive or negative association, and linear association.
4) Using a piece of spaghetti or wire, determine the line of best fit and draw it on the scatterplot. Describe how you determined where to place the line and why that line best fits the data.
5) Determine the equation of the line you drew in step 4. Interpret its slope and *y*-intercept in the context of the data.
6) Describe how well the points follow the line you made in step 4.
7) Predict the student/teacher ratio for a district whose teachers make an average of $57,000 a year.
8) Give at least two reasons why straight lines are used to model relationships between two quantitative variables with a linear association.

Answer:

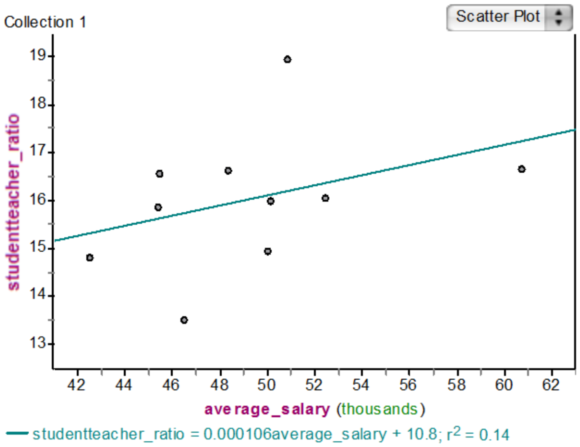
1)



2) This is real data about teacher salary and student/teacher ratios. There are many factors that influence teacher salary and student/teacher ratio, so they aren't expected to follow a perfect line.

3) There is a positive association between average salary and student/teacher ratio, meaning that as average salary of teachers in a district increases so does the student/teacher ratio. The last point stands out as unusual. That district had a much higher average teacher salary than the rest.

4) Answers will vary. The plot below displays the least-squares regression line.



5) Answers will vary. For the least squares line, y = 0.000106x +10.8. For every $10,000 increase in a district's average teacher salary, the student/teacher ratio increases by approximately 1 on average. If the average teacher salary in a district is 0, the student/teacher ratio is 10.8.

6) The points have a lot of scatter about the line. There are many points that are relatively far from the line. The association of the points is pretty weak.

7) 16.842 student/teacher ratio.

8) If the data show a linear association, then the linear model can show the general trend in the data and be used for making predictions.

Possible Extensions

Students can start to explore data sets that have:

a) Multiple linear trends to them, so are best fit through multiple lines of best fit.
b) Curvature that can be straightened out through linearization techniques (like exponential, which can be transformed to be linear by plotting log (*y*) vs. *x*) after which linear models can be found. See Peck, Olsen, & Devore (2011) for explanation of such techniques.

References

1. Casey, S., & Kaplan, J. (under review). *Statistical knowledge for teaching: The case of informal line of best fit*.

2. Casey, S., & Wilson, D. (2012). *Student conceptions of the line of best fit*. Research proposal.

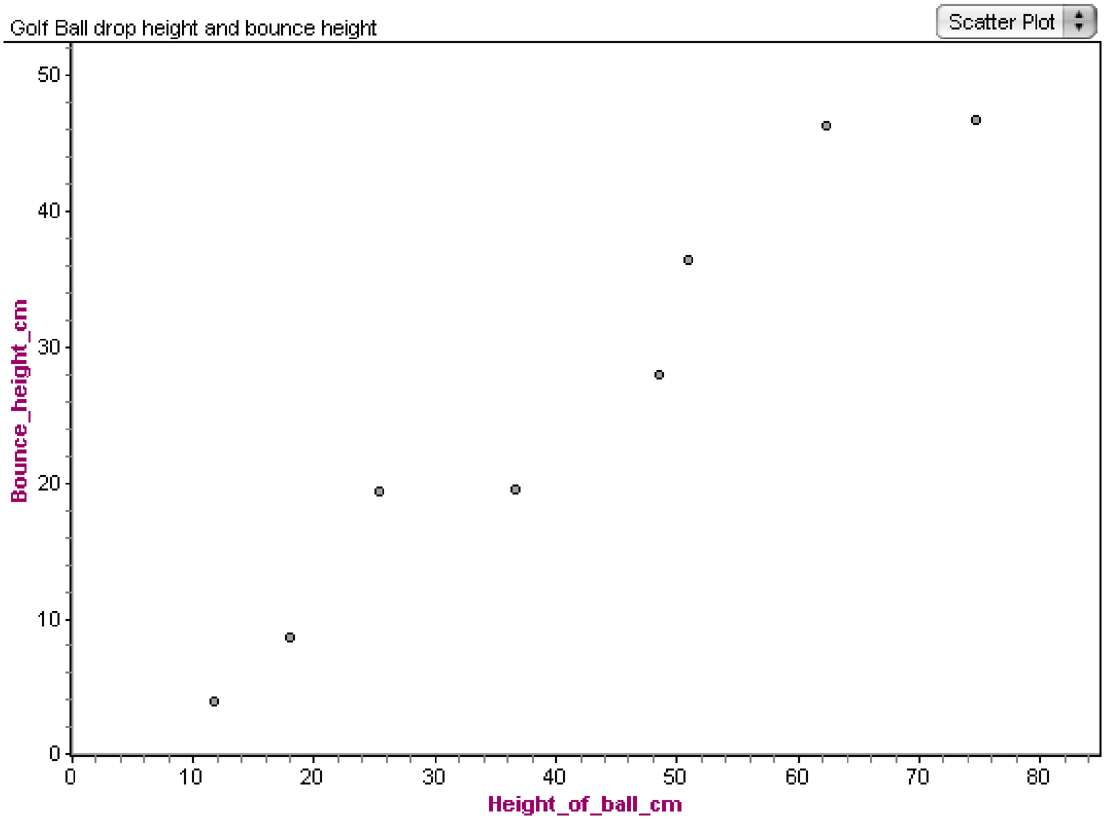
3. Franklin, C., Kader, G., Mewborn, D., Moreno, J., Peck, R., Perry, M., & Scheaffer, R. (2007). *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report: A Pre-K–12 Curriculum Framework*. Alexandria, VA: American Statistical Association.

4. Peck, R., Olsen, C., & Devore, J. (2011). *Introduction to Statistics and Data Analysis* (4th edition). Boston, MA: Brooks/Cole.

What Fits? Activity Sheet

Introduction

An eighth grade class was investigating how high a golf ball bounces when it is dropped from different heights. Students were given eight set heights to drop the ball from. Then they dropped a golf ball from each of those heights and measured how high the ball bounced back up. Below is the scatterplot they made of their data.



Part One Instructions

Determine where you think the line of best fit should be placed on the scatterplot above by manipulating a piece of spaghetti or wire.

Draw your line on the scatterplot once you have determined its best placement.

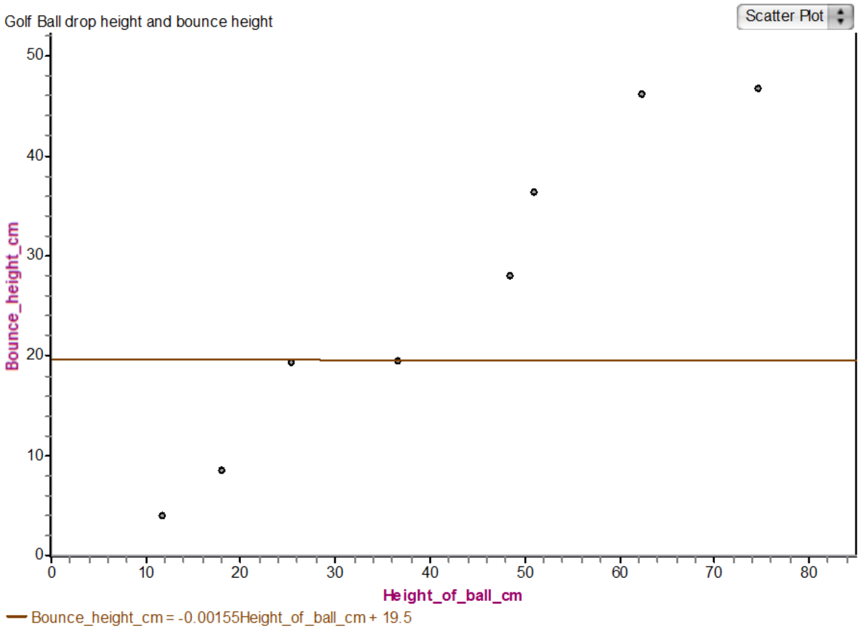
Use your line to predict the height the golf ball will bounce to if it is dropped from 60 centimeters. Is the prediction reasonable?

Part Two Instructions

In the class that collected the data, the teacher gave the students a copy of the scatterplot and asked each of them to find the line of best fit for the data. The teacher then asked each student to explain why they put the line where they did, defining the criterion for the line of best fit. Some sample student responses to this task are on the following pages (pages 11-13). For each student's response:

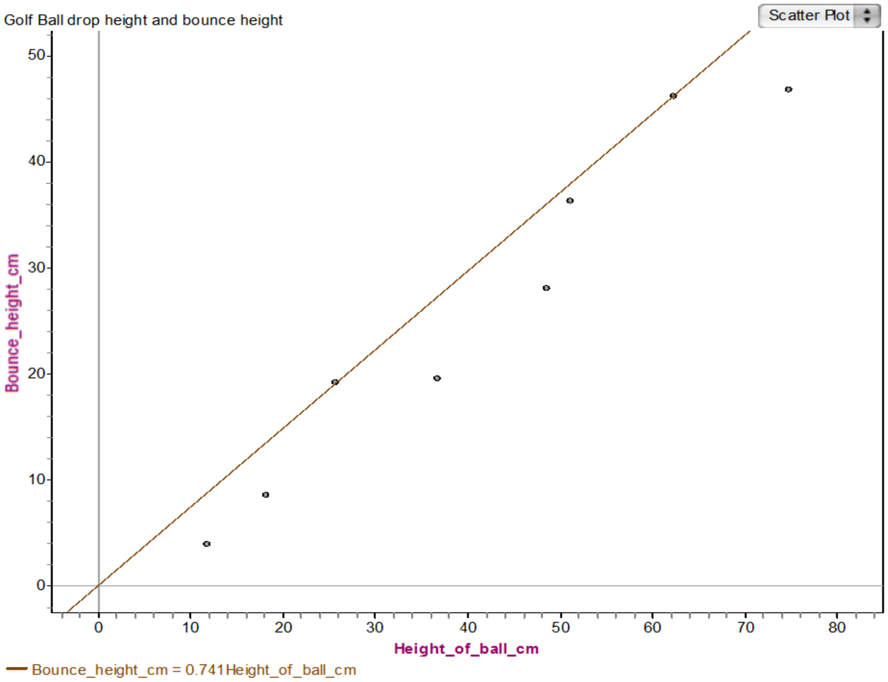
1) Describe what the student's line of best fit tells them about the relationship between the bounce height of a ball and the height it was dropped from.
2) Interpret the slope and *y*-intercept of the student's line in the context of this data set. Do their interpretations seem reasonable in this context?
3) Use the student's line to predict the height the golf ball will bounce to if it dropped from 60 centimeters. Is the prediction reasonable?
4) Analyze the student's criterion. Will the criterion always work to produce a line that accurately models any data set? If it will, explain why. If it won't, draw at least one example of a scatterplot with the line placed using that criterion and explain why the criterion produces a bad line of best fit.

Christina's response



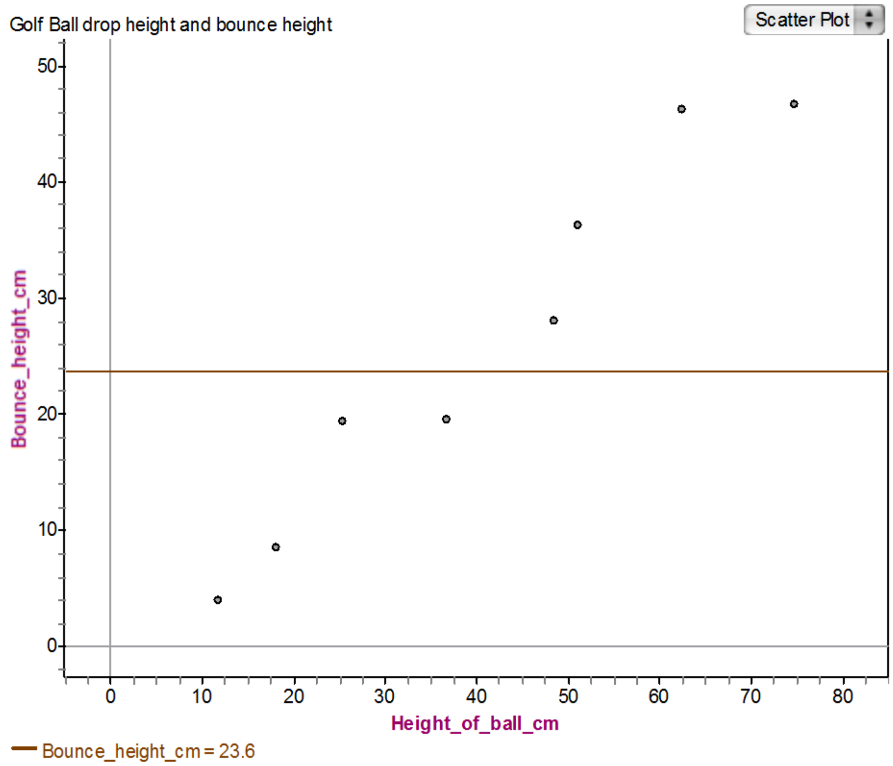
I thought of the line of best fit like the mode, so I put the line through two points with the same *y*-coordinate because they occur most often.

Edward's response



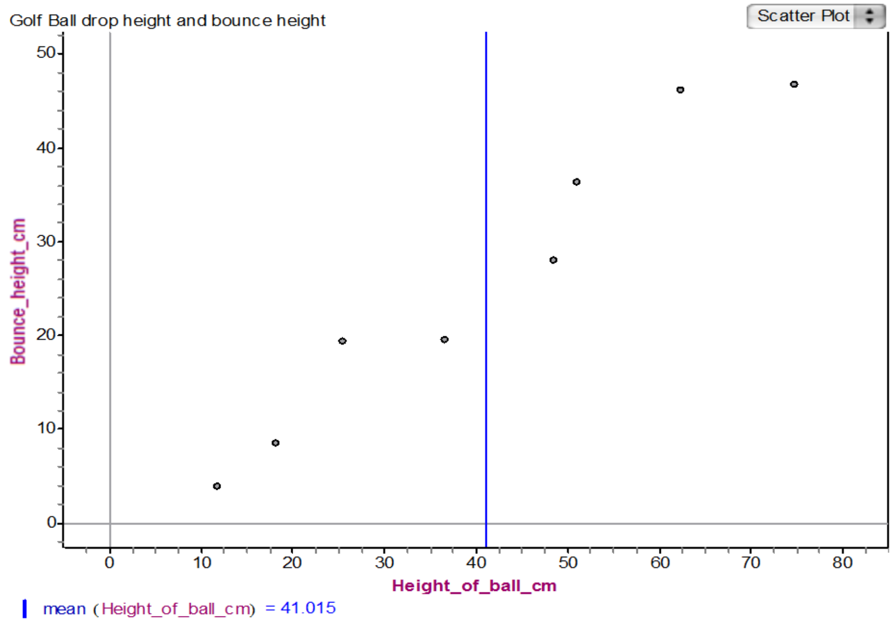
The line needs to start at (0,0) then go through the most dots. I got my line to go through two of the dots so I put it there.

Molly's response



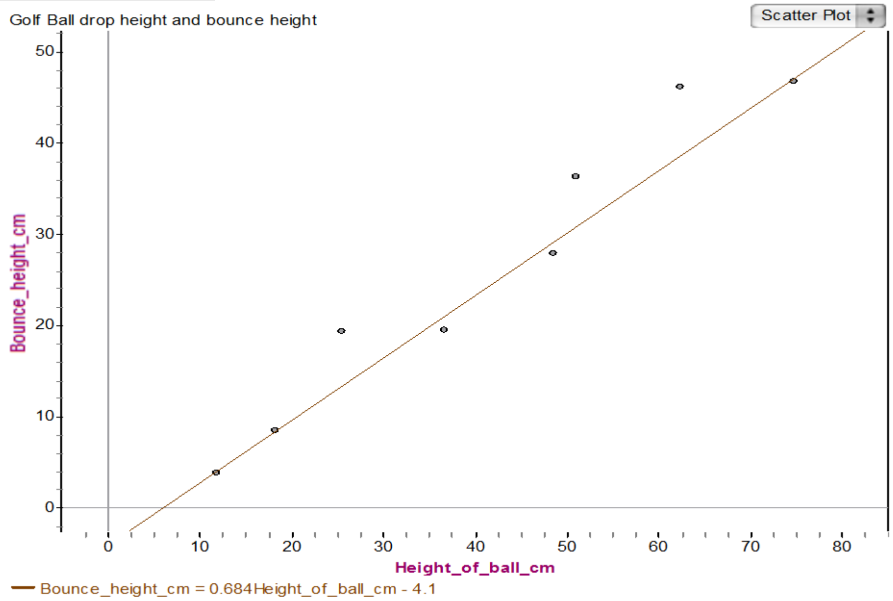
The line should be in the middle of the highest and lowest points because that's like the average.

Petra's response



I know the line should go in the middle of the data. I put it here so it would be in the middle, four points on each side.

Talia's response



I tried to make my line go through the most dots.

Summary Questions

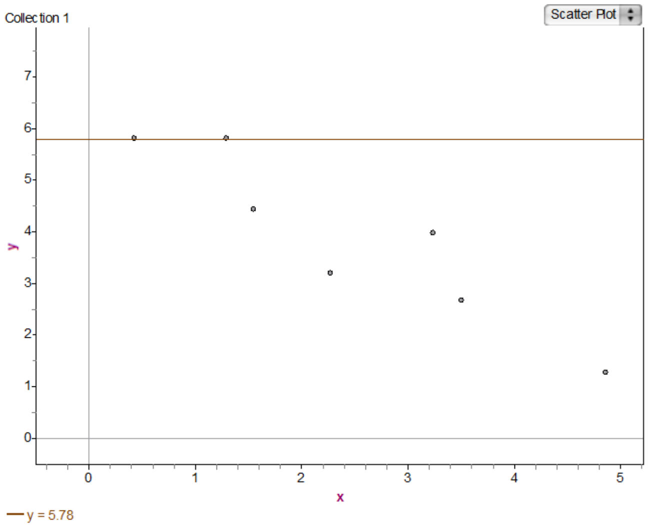
1) What is the best criterion to use when determining where the line of best fit is for any data set? Justify your response.
2) Another student asks you "What *is* the line of best fit?". Write your response to this student.

Answers to Part I Discussion Questions
This activity will be driven by the teacher-directed question: **"What is the relationship between the drop height of a golf ball and the height of its first bounce?"** Teachers should conduct a discussion using the following questions:

- What is our population of interest?
  The population of interest is all golf balls of this type.
- What are the two variables of interest?
  The drop height and height of first bounce of a golf ball.
- Which variable is the explanatory variable? Why?
  The drop height is the explanatory variable because it explains how high the ball bounces.
- Do you expect there to be a strong association between these two variables? Why?
  A strong association is expected as long as the data is collected accurately because there should be a relationship between the height the ball is dropped from and how high it bounces.
- What form do you expect the relationship to follow (i.e., what function family would be used to model this relationship)? Why?
  A linear model would make sense because I expect the bounce height to increase at a constant rate.
- What direction do you expect the association to take? Why?
  It should have a positive association since it will bounce higher when it is dropped from higher heights.
- Do you expect a golf ball to bounce back up to its drop height? Lower than its drop height? Higher than its drop height?
  The golf ball will likely bounce up to lower than its drop height.

Answers to Activity Part Two
Below are possible correct responses.
Christina

1) The ball bounces to the same height no matter how high the ball is dropped from.
2) The bounce height of the ball goes down by 0.00155 cm on average when the drop height is increased by 1 cm. This doesn't make sense because it should get higher as you drop the ball from higher heights.
   The ball would bounce to 19.5 cm if it wasn't dropped. This doesn't make sense because it should not bounce if it's not dropped, so the bounce height should be zero.
3) 19.4 cm. This is not reasonable because the bounce height for 60 should be between 38 cm and 48 cm, the bounce heights for drop heights of 52 and 65 cm.
4) No, this method won't usually work. Below is a scatterplot where the first two points have the same *y*-value of 5.8. Putting the line through those points produces a line that is horizontal, but the points overall have a clearly negative trend so the line doesn't fit the data well.

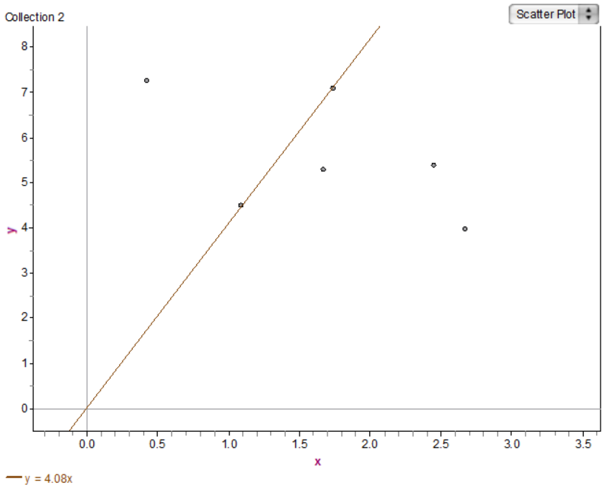


Edward

1) The line shows the bounce height of the ball increases as the drop height increases.
2) The bounce height of the ball goes up by 0.741 cm on average when the drop height is increased by 1 cm. This is reasonable because the bounce height increases when the drop height increases, and I would expect the bounce height to increase by less than 1 cm if you increase the drop height by 1 cm.

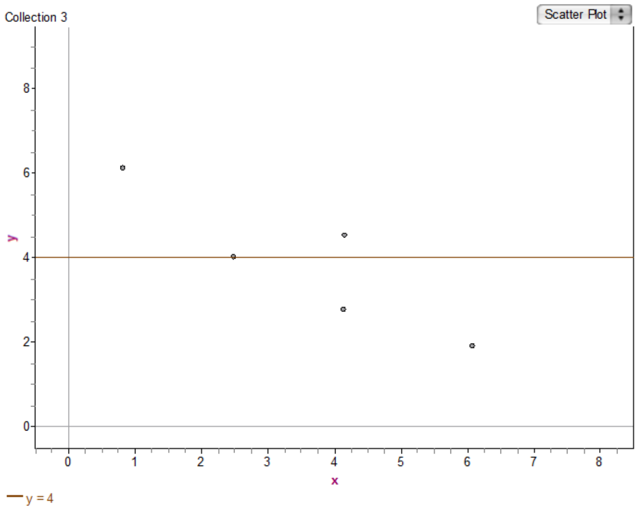
   The ball would bounce to 0 cm if you dropped it from 0 cm. This makes sense because the ball shouldn't bounce if you don't drop it!
3) 44.46 cm. That is reasonable because it's near where the data points for drop heights of 52 and 65 cm are.
4) It makes sense to have the line go through (0,0) here because of the context about ball bounce and drop heights, but it won't make sense in every context. Also, going through the most dots won't always work because it's possible the dots that line up aren't in the middle of the cloud of points.

   Here's an example of a scatterplot where Edward's criteria doesn't work well. Since the data have a negative association, it doesn't make sense to make the line start at (0,0). Also the line through the most points is going up, when it needs to be going down.



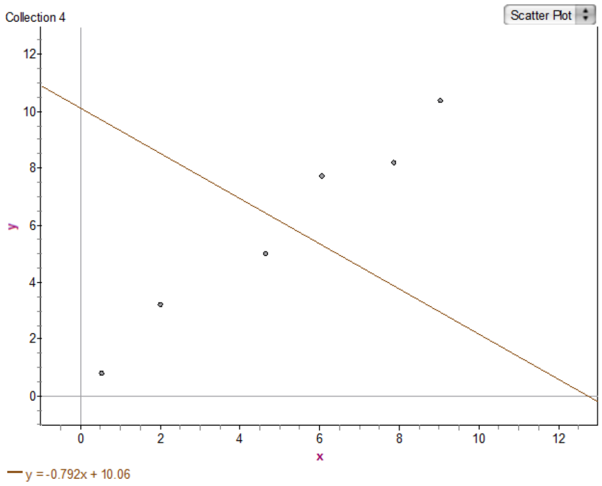
Molly

1) The ball bounces to the same height no matter how high the ball is dropped from.
2) This line has a slope of zero, which means that if the ball is bounced from higher heights the bounce height doesn't change. This doesn't make sense because the ball should bounce higher if it's dropped from higher heights.
   The ball would bounce to 23.6 cm if it wasn't dropped. This doesn't make sense because it should not bounce if it's not dropped, so the bounce height should be zero.
3) 23.6 cm. That's not reasonable because it's less than the bounce height of a ball dropped from 50 cm, and it should be greater.
4) The student's idea that the line should be in the middle is fine, but the line needs to be in the middle of all the points not just the highest and lowest. It also needs to have the same angle as the data, not horizontal every time.
   The scatterplot below shows another example of why this criterion is bad.



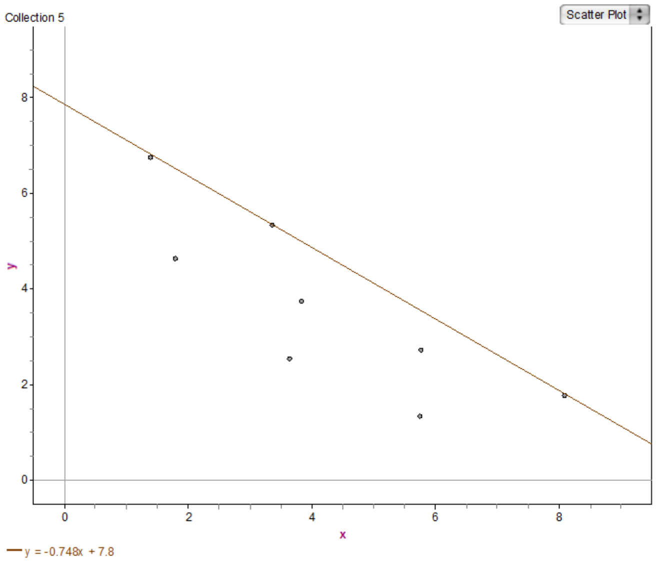
Petra

1) The line says the drop height of the ball is always the same for every bounce height.
2) This line's slope is undefined, which means that the drop height of the ball is always 41.105 cm. This doesn't make sense because the ball was and can be dropped from a bunch of different heights, not just 41.105 cm.
   The line doesn't have a *y*-intercept.
3) The line can't be used to predict the bounce height.
4) Like Molly, Petra has the right idea that the line should go in the middle of the data, and even the idea that there's the same number of points on each side of the line can work sometimes. But the scatterplot below shows that just having that for the criterion doesn't make a good line of best fit.



Talia

1) The line shows the bounce height of the ball increases as the drop height increases.
2) The bounce height of the ball goes up by 0.684 cm on average when the drop height is increased by 1 cm. This is reasonable because the bounce height increases when the drop height increases, and I would expect the bounce height to increase by less than 1 cm if you increase the drop height by 1 cm.
   The ball should bounce -4.1 cm if dropped from height zero, which means the ball isn't dropped. This is not reasonable because a ball can't bounce a negative height.
3) 36.94 cm. I think this prediction is too low. That's about the height the ball bounced when dropped from 52 cm and it should bounce higher than that if it's dropped from 8 cm higher.
4) Going through the most points can make a good line of best fit but it won't always. The scatterplot below shows a case where it makes a bad line of best fit. The line is too high.



Possible Answers to Summary Questions

1) I think the best criterion is the one that makes the total distances of the points above and below the line the same. That way it gets the line in the middle and follows the trend of the data set.
2) It's a line that shows your best guess as to where the data would be for those values that you don't have.