Armspans

Debra L. Hydorn
University of Mary Washington
dhydorn@umw.edu

Published: August 2012

Overview of Lesson

In this activity students conduct an investigation to compare the arm span lengths of the boys and girls within their class. They will construct box plots and calculate measures of center and spread to determine if there is a difference between boys and girls. They will also compare the proportion of boy's and girl's arm span lengths that are within one standard deviation of their group mean. After observing the arm spans of their classmates, students can use the Census at School website to take random samples of boys and girls from schools participating in the Census at School program. Samples from the Census at School data base are downloaded in a csv format that can be transferred to a data analysis program such as Excel.

GAISE Components

This investigation follows the four components of statistical problem solving put forth in the *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report*. The four components are: formulate a question, design and implement a plan to collect data, analyze the data by measures and graphs, and interpret the results in the context of the original question. This is a GAISE Level B activity.

Common Core State Standards for Mathematical Practice

3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.

Common Core State Standards Grade Level Content (Grades 6 through 7)

6. SP. 1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.
6. SP. 2. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
6. SP. 4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
7. SP. 1. Understand that statistics can be used to gain information about a population by examining a sample of the population.

7. SP. 3. Informally assess the degree of visual overlap of two numerical distributions with similar variability, measuring the difference between the centers by expressing it as a multiple of a measure of variability.

7. SP. 4. Use measures of center and measures of variability for numerical data form random samples to draw informal comparative inferences about two populations.

NCTM Principles and Standards for School Mathematics

Data Analysis and Probability Standards for Grades 6-8

   Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them:

- formulate questions, design studies, and collect data about a characteristic shared by two populations or different characteristics within one population;
- select, create, and use appropriate graphical representations of data, including histograms, box plots, and scatterplots.

   Select and use appropriate statistical methods to analyze data:

- find, use and interpret measures of center and spread, including mean and interquartile range;
- discuss and understand correspondence between data sets and their graphical representations, especially histograms, stem-and-leaf plots, box plots and scatterplots.

   Develop and evaluate inferences and predictions that are based on data

- use observations about differences between two or more samples to make conjectures about the populations from which the samples were taken;
- use conjectures to formulate new questions and plan new studies to answer them.

Prerequisites

Students should have prior experience collecting data and calculating measures of center and spread and constructing box plots of quantitative data.

Learning Targets

Students will interpret box plots that compare two groups. They will also interpret and compare measures of center and spread.

Time Required

One class period.

Materials Required

Tape measures for measuring arm span lengths in centimeters. Computer(s) for accessing the Census at School website. A copy of the Activity Sheet (pages 9 and 10) for each student.

Instructional Lesson Plan

The GAISE Statistical Problem-Solving Procedure

I. Formulate a Question

Begin the lesson by asking the students to consider in what situations arm span lengths are important and if they think that boys and girls have the same arm spans. Have them guess what the distribution of arm spans might look like for the whole class and whether the distribution of arm span lengths for boys and girls would be the same. Encourage the students to consider situations where having a longer arm span would be an advantage.

Possible questions to ask:
1. How is arm span length defined?
2. Can someone measure their own arm span length?
3. How might measurement consistency be an issue with this data? What approaches can be made to develop measurement consistency?
4. Are the arm span lengths of boys and girls the same or different?
5. What methods can we use to compare the arm span lengths of boys and girls?
6. What are some other factors besides gender that might affect arm span lengths? (Possible factors that could be studied using the Census at School data base are age, state, and year the data was collected.)

II. Design and Implement a Plan to Collect the Data

Have the students determine a process of measuring arm span lengths to the nearest centimeter and then record the arm span lengths for the class. If enough tape measures are available and an appropriate protocol for measuring arm spans lengths is determined, students could be divided into pairs and asked to measure and record the arm span length of their partner.

III. Analyze the Data

There are various ways to analyze the collected data. For example, the class can calculate measures of center and spread for the boys and girls in their class. Box plots can be constructed from the class data to compare boys and girls. Table 1 provides some example data. For this data set, samples were requested from all states and all years available for students in grade 6 from the Census at School Random Sampler data base
http://www.amstat.org/censusatschool/RandomSampleForm.cfm .

Table 1. Example class data.

Boy's Arm Spans (cm)	Boy's Arm Spans (cm)		Girl's Arm Spans (cm)	Girl's Arm Spans (cm)
149	155		163	162
140	158		161	158
164	149		148	164

155	158		149	161
150	150		142	163
137	146		144	145
143	152		139	162
155			175	

Some descriptive statistics for this data are provided in Table 2.

Table 2. Descriptive Statistics for Example Arm Span Lengths (cm).

	Mean	Median	Range	Standard Deviation
Boys	150.7	150	27	7.2
Girls	155.7	161	36	10.4

For this example data set girls have higher measures of center and greater measures of spread. Box plots for the two groups are shown in Figure 1.
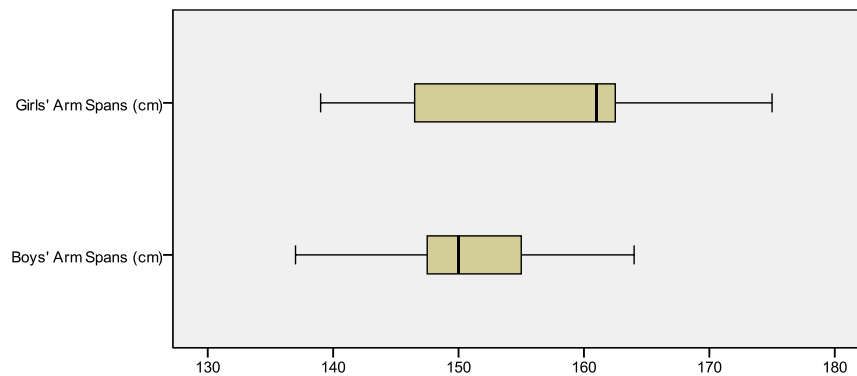

Figure 1. Comparative box plot for example data.

The distributions have roughly the same shapes; both are fairly symmetric. The distribution for the girls has a larger range and a larger median. The middle 50% of the boy's distribution is contained within the middle 50% of the girl's data.

The sorted values for this data are:

      Boys: 137, 140, 143, 146, 149, 149, 150, 150, 152, 155, 155, 155, 158, 158, 164

      Girls: 139, 142, 144, 145, 148, 149, 158, 161, 161, 162, 162, 163, 163, 164, 175

The intervals within one standard deviation of the mean for each group are:

      Boys: $150.7 - 7.2$ to $150.7 + 7.2$ or 143.5 cm to 157.9 cm

      Girls: $155.7 - 10.4$ to $155.7 + 10.4$ or 145.3 cm to 166.1 cm

Nine out of fifteen of the boys are within one standard deviation of their mean and ten out of fifteen of the girls are within one standard deviation of their mean. While the girls have a greater amount of spread the proportion of boys and girls within one standard deviation of the mean of their group is similar.

IV. Interpret the Results

In addition to interpretive points made in analyzing the data, the students can be prompted to generate and answer further data analysis questions such as:

- On average, how much do the arm spans of boys and girls differ? How much do they vary?
- What percentage of the boys is within one standard deviation of the mean for their data? What percentage of the girls is within one standard deviation of the mean for their data?
- What generalizations, if any, can we make using the results from this data?

Assessment

Suppose that Jeremy's class produced the following results when performing this experiment:

Our Class Data: Arm Span Lengths (cm)		
Karen	153	F
Joshua	144	M
Jamal	141	M
Trevor	151	M
Alicia	155	F
John	135	M
Bob	133	M
Barb	148	F
Kathi	142	F
Joannie	157	F
Mary	163	F
Lee	155	M
Jamie	155	F
Leilani	148	F
Reid	141	M

Use the approaches below to compare the arm span lengths of the boys and girls in this class.

(a) Find the mean, median, range and standard deviations for the arm span lengths of the boys in this class. Find the mean, median, range and standard deviation for the arm span lengths of the girls in this class. Compare the two distributions using these measures of center and spread. Which group has a larger arm span length on average? Which group varies more?

(b) Sort the data within each group then determine what proportion in each group is within one standard deviation of that group's mean. Are the proportions similar?

(c) Produce box plots of the boys and girls data. Compare the distributions of arm span lengths.

Answers

(a) The measures of center and spread are provided in the table below:

	Mean	Median	Range	Standard Deviation
Boys	142.9	141	22	8.0
Girls	152.6	154	21	6.5

The girls have a larger mean. On average the girl's arm span lengths are about 10 cm longer. However, within each group the mean and medians are close in value. This suggests that the distributions of arm span lengths are fairly symmetric. The two groups have similar amounts of variability; the ranges differ by just 1 cm and the standard deviations differ by 1.5 cm.

(b) The sorted data is provided below:

   Boys: 133, 135, 141, 141, 144, 151, 155

The interval within one standard deviation of the mean is $142.9 - 8$ to $142.9 + 8$ or 134.9 cm to 150.9 cm. Four of the seven boys have an arm span length that is within one standard deviation of the mean.

   Girls: 142, 148, 148, 153, 155, 155, 157, 163

The interval within one standard deviation of the mean is $152.6 - 6.5$ to $152.6 + 6.5$ or 146.1 cm to 159.1 cm. Six of the eight girls in this class have an arm span length that is within one standard deviation of the mean.

(c) The box plots are shown below:
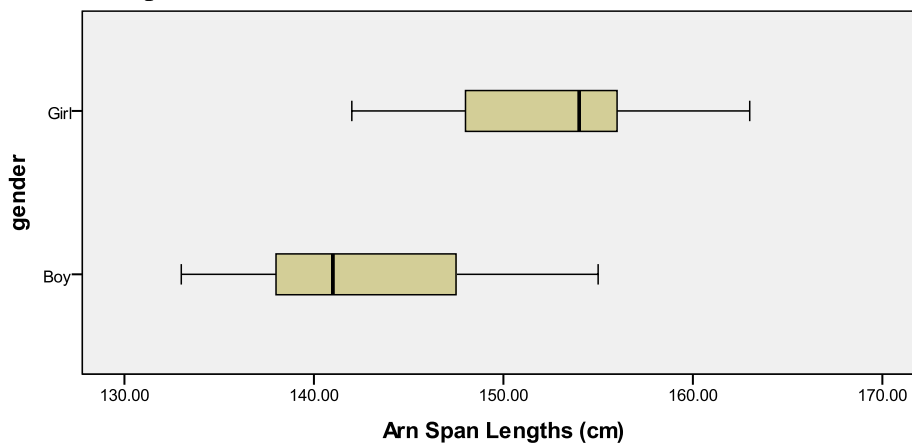
The box plots have a similar shape. They are both symmetric with approximately the same range and interquartile range. Each of the values of the five number summary used to create the box plot for the boys is below the corresponding five number summary value for the girls. This suggests that on average the boys in this class have a smaller arm span length.

Possible Extensions

1. To investigate the distribution of arm span lengths, larger samples could be taken using classes at the same grade level. This could also be accomplished using the **Census at School Random Sampler** http://www.amstat.org/censusatschool/RandomSampleForm.cfm. This data base consists of survey information completed by the students of teachers who have registered their classes with Census at School. The Census at School Random Sampler allows anyone to access this data base to select random samples according to a variety of factors (e.g., age, gender, and state). One of the responses provided by the participating students is their arm span length to the nearest cm. The Random Sampler has options for taking samples of 10, 25, 50, 100, 250 or 500 and samples can be drawn from one state, a collection of states or all states combined. Note that, as with any survey, some students will have left the arm span question blank and some of the data values are suspect (e.g., an arm span length of 1000 cm). Using the random sampler, students can select a sample of boys and a separate sample of girls then record the arm span lengths for their two samples. Data from the Sampler is downloaded as a csv file which can be opened in Excel. Larger samples can be copied and pasted in a spreadsheet or the data editor of a data analysis program. After examining the results for their sample (e.g., determine the difference in means or medians, compare the amount of variability) students could then compare their results to their classmates' or to the results of the population of students who provided their information to Census at School. Note: In case sampling from the Census at School Random Sampler is not available a collection of ten different samples of 30 from the data base are provided below:

Data set 1:
Boys: 162, 166, 153, 182, 145, 154, 153, 100, 144, 162, 154, 149, 140, 156, 128, 140, 103, 182, 172, 190, 174, 163, 140, 148, 157, 163, 155, 150, 163, 150

Girls: 150, 175, 163, 130, 149, 160, 148, 142, 157, 156, 157, 160, 150, 148, 152, 152, 156, 160, 163, 166, 147, 143, 149, 164, 159, 157, 155, 149, 170, 164

Data set 2:
Boys: 169, 146, 154, 166, 154, 158, 152, 145, 151, 168, 175, 139, 150, 159, 142, 142, 152, 155, 171, 157, 139, 157, 152, 163, 140, 156, 85, 150, 150, 145

Girls: 136, 149, 162, 134, 149, 161, 167, 146, 152, 143, 165, 167, 162, 170, 146, 160, 125, 160, 169, 157, 116, 167, 153, 146, 158, 156, 157, 148, 161, 155

Data set 3:
Boys: 178, 81, 169, 160, 138, 145, 150, 164, 165, 146, 156, 157, 180, 166, 146, 161, 157, 130, 155, 108, 163, 163, 170, 135, 155, 146, 160, 146, 86, 159

Girls: 145, 101, 162, 170, 156, 156, 140, 147, 152, 151, 100, 160, 90, 148, 156, 175, 143, 160, 158, 168, 141, 150, 158, 166, 153, 147, 161, 151, 141, 157

Data set 4:

Boys: 169, 158, 140, 142, 144, 159, 151, 144, 148, 150, 144, 130, 152, 152, 146, 157, 142, 143, 160, 154, 169, 150, 158, 153, 180, 145, 158, 150, 146, 180

Girls: 153, 81, 179, 153, 103, 102, 171, 170, 90, 143, 151, 148, 150, 130, 164, 155, 153, 169, 154, 169, 85, 105, 100, 149, 152, 156, 164, 152, 148, 160

Data set 5:
Boys: 170, 148, 174, 155, 152, 168, 158, 163, 149, 160, 160, 157, 168, 165, 153, 164, 140, 148, 135, 153, 154, 167, 125, 163, 172, 158, 139, 162, 152, 169

Girls: 154, 147, 161, 162, 152, 155, 161, 154, 160, 160, 153, 165, 152, 150, 80, 147, 150, 143, 170, 150, 156, 110, 152, 152, 157, 144, 152, 167, 148, 150

Data set 6:
Boys: 146, 144, 143, 166, 154, 166, 135, 166, 147, 150, 142, 149, 166, 152, 150, 132, 166, 149, 154, 164, 150, 142, 144, 150, 157, 156, 157, 148, 150, 165

Girls: 137, 151, 175, 150, 80, 155, 123, 165, 144, 86, 103, 144, 159, 176, 143, 158, 157, 157, 144, 161, 150, 169, 158, 168, 151, 148, 195, 168, 166, 100

Data set 7:
Boys: 163, 148, 160, 144, 140, 171, 100, 120, 168, 140, 154, 150, 155, 150, 169, 159, 140, 149, 143, 160, 150, 125, 160, 160, 161, 156, 142, 158, 159, 120

Girls: 152, 148, 160, 122, 80, 149, 172, 168, 155, 150, 165, 160, 156, 176, 156, 120, 152, 139, 139, 150, 159, 162, 163, 155, 135, 160, 150, 135, 166, 139

Data set 8:
Boys: 163, 144, 156, 149, 157, 156, 136, 171, 141, 154, 134, 160, 167, 152, 84, 153, 157, 158, 143, 158, 167, 135, 156, 170, 141, 145, 148, 148, 170, 160

Girls: 151, 156, 146, 155, 142, 142, 157, 152, 165, 145, 169, 180, 163, 154, 152, 162, 141, 160, 81, 153, 159, 149, 166, 150, 152, 137, 142, 135, 156, 165

Data set 9:
Boys: 138, 145, 161, 130, 150, 158, 147, 100, 145, 164, 110, 179, 152, 160, 163, 160, 146, 159, 178, 170, 140, 167, 164, 102, 143, 162, 155, 149, 155, 157

Girls: 164, 148, 156, 151, 160, 150, 165, 175, 140, 159, 180, 150, 150, 155, 134, 152, 155, 150, 163, 170, 158, 160, 165, 154, 141, 159, 152, 150, 140, 107

Data set 10:
Boys: 81, 140, 145, 155, 165, 172, 151, 145, 80, 158, 170, 161, 149, 137, 163, 140, 159, 163, 158, 162, 200, 116, 148, 152, 163, 162, 169, 148, 151, 148

Girls: 159, 141, 156, 163, 158, 154, 93, 158, 147, 117, 142, 169, 166, 158, 152, 145, 150, 100, 162, 165, 156, 152, 145, 159, 157, 145, 166, 86, 135, 169

2. This activity could be extended to compare students at different grade levels. Up to about age 12 there is no difference between the arm span lengths and heights of boys and girls. But at age

13 boys become significantly taller with longer arm spans.   The Census at School Random Sampler could be used for this activity or data from classes at different grade levels could be obtained.

References

Adapted from the Armspans Census at School New Zealand activity created by Julia Horring, revised by Lisa Gilmore http://www.censusatschool.org.nz/classroom-activities/armspans/.

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Armspans Activity Sheet

1. What are some questions we should ask to determine someone's arm span length?

2. Record your arm span length: _____ cm

3. Record the results for the class in the following table:

Name	Arm Span Length (cm)	Boy or Girl?	Name	Arm Span Length (cm)	Boy or Girl?

4. Organize the arm span lengths by gender.

Boys:

Girls:

5. Sort the arm span lengths for each group from smallest to largest and then find the mean, median, range and standard deviation:

Boys:

Girls:

6. Using the sorted data, find the values needed to construct box plots of the data:

Boys:

Girls:

7. Construct the box plots:

8. Compare the distributions of arm span lengths for boys and girls.

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