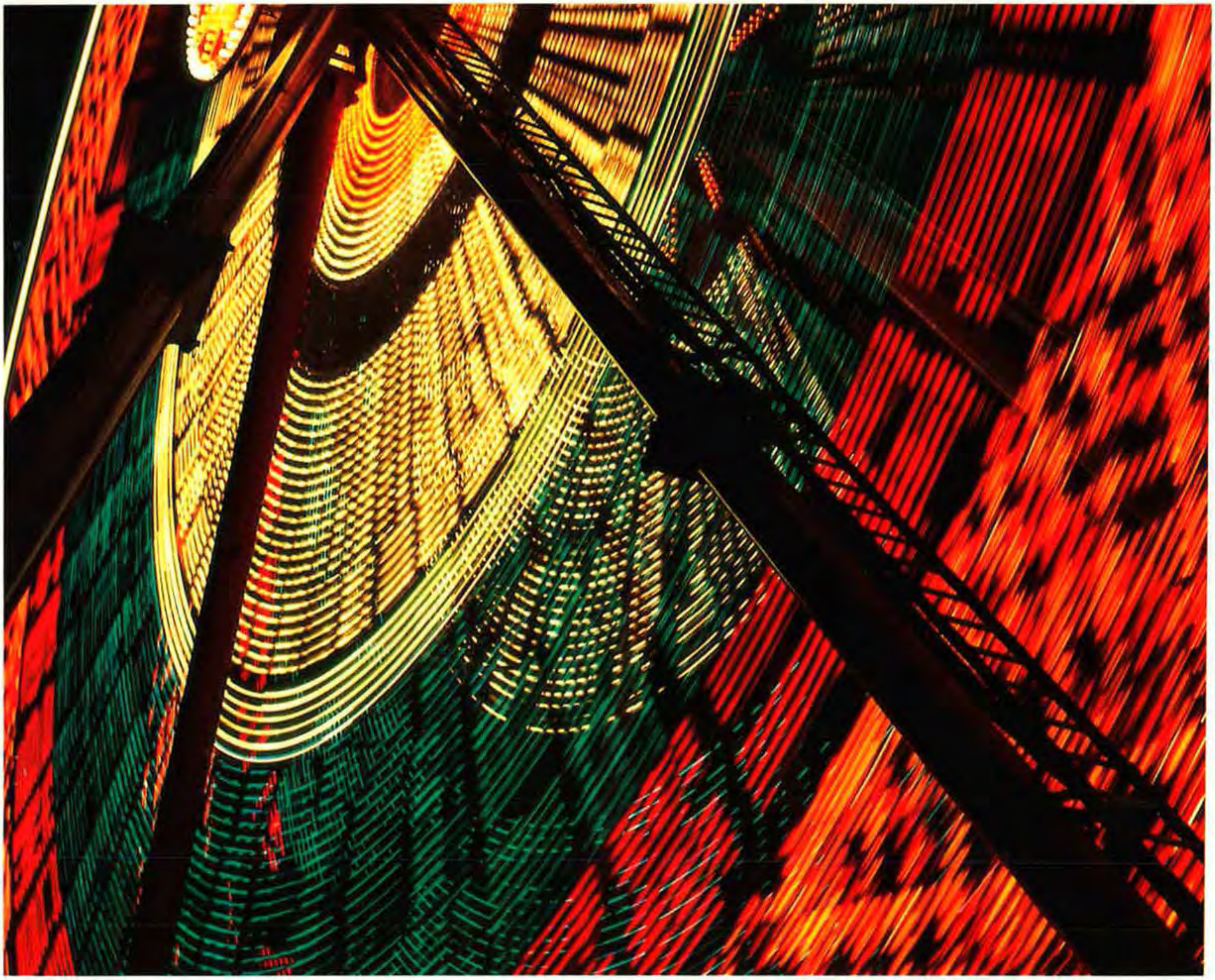
INTRODUCTORY ALGEBRA

TEACHER'S EDITION

# Mathematics in a World of Data

BURRILL, CLIFFORD, ERRTHUM, KRANENDONK, MASTROMATTEO, O'CONNOR

DATA-DRIVEN MATHEMATICS



DALE SEYMOUR PUBLICATIONS®

# Mathematics in a World of Data

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## DATA-DRIVEN MATHEMATICS

Jack Burrill, Miriam Clifford, Emily Errthum, Henry Kranendonk,
Maria Mastromatteo, and Vince O'Connor

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# Table of Contents

# About *Data-Driven Mathematics*

Historically, the purposes of secondary-school mathematics have been to provide students with opportunities to acquire the mathematical knowledge needed for daily life and effective citizenship, to prepare students for the workforce, and to prepare students for postsecondary education. In order to accomplish these purposes today, students must be able to analyze, interpret, and communicate information from data.

*Data-Driven Mathematics* is a series of modules meant to complement a mathematics curriculum in the process of reform. The modules offer materials that integrate data analysis with secondary mathematics courses. Using these materials helps teachers motivate, develop, and reinforce concepts taught in current texts. The materials incorporate the major concepts from data analysis to provide realistic situations for the development of mathematical knowledge and realistic opportunities for practice. The extensive use of real data provides opportunities for students to engage in meaningful mathematics. The use of real-world examples increases student motivation and provides opportunities to apply the mathematics taught in secondary school.

The project, funded by the National Science Foundation, included writing and field testing the modules, and holding conferences for teachers to introduce them to the materials and to seek their input on the form and direction of the modules. The modules are the result of a collaboration between statisticians and teachers who have agreed on the statistical concepts most important for students to know and the relationship of these concepts to the secondary mathematics curriculum.

A diagram of the modules and possible relationships to the curriculum is on the back cover of each Teacher's Edition of the modules.

# Using This Module

## Why the Content Is Important

This module establishes a foundation that will allow and encourage students to acquire the tools necessary to harness the power of mathematics and statistics in their own lives. By looking at information presented by the media and discovering patterns in activities and simple experiments, students begin to understand the need for quantitative information in decision making. In the lessons, students examine numbers in context, data. They learn an investigation approach to help them use data to answer questions or address problems. Skills such as percent concepts and constructing graphs and plots are used in the context of real-world applications. The focus is on problem solving in cooperative groups where students use mathematical and statistical tools to better understand their world.

*Mathematics in a World of Data* is divided into four units.

### Unit I: Context and Units

Have students discussed how decisions are made about which United States cities are the best places to live? How much money do students think people in different regions of the country spend on soft drinks each year? Data can be collected and studied to answer these questions. Mathematical tools used in this section for working with data include: organizing data in tables or charts, creating plots or graphs, calculating averages, and studying the variability in data.

### Unit II: Sorting and Counting

Data come in many forms, and some, such as the colors of M&Ms® in a package, are not numbers at all. Categorical data and measurement data are explained in Lesson 4. They are summarized in different ways. Data are often used to answer questions and support arguments. An investigation process is developed in Lesson 5 to provide a structured approach to problem solving. This process is used in Lesson 6 to help students sort, count, and represent data involving M&Ms.

### Unit III: Using Percents

Percents are used to compare groups or populations of different sizes. In this unit, students compare area and population of the continents on 10-by-10 grids that provide a visual representation for percent. Percents are used in reporting the results of a telephone poll and interpreting data on a shot chart for high-school basketball players. Students also use percents to summarize cup-tossing data collected in Lesson 11.

### Unit IV: Measurement Data in Experiments

Measurement data are used to measure observations, results, or outcomes. Measurement data can be represented on the real-number line and can be summarized by finding a center. The activities and experiments in this section all use measurement data. The experiments include measuring reaction time and finding the average size of bubbles that can be blown with a straw.

### Content

Mathematics content: Students will be able to

- Count, sort, order, and compare rational numbers.
- Compute with positive rational numbers.
- Compute with percent.
- Work with measurement.
- Understand concepts of inequality.
- Understand concepts of probability.
- Use estimation techniques.

Statistics content: Students will be able to

- Work with categorical and measurement data.
- Collect data by observation, survey, and experiments.
- Construct frequency and cumulative frequency distributions.
- Represent data in tables, bar graphs, number-line plots, stem-and-leaf plots, scatter plots, and box plots.
- Make predictions.
- Understand concepts of variability, bias, and association.

### Instructional Model

The instructional emphasis in *Mathematics in a World of Data,* as in all of the modules in *Data-Driven Mathematics,* is on discourse and student involvement. Each lesson is designed around a problem or mathematical situation and begins with a series of introductory questions or scenarios that can prompt discussion and raise issues about that problem. These questions can involve students in thinking about the problem and help them understand why such a problem might be of interest to someone in the world outside the classroom. The questions can be used in whole-class discussion or in student groups. In some cases, the questions are appropriate to assign as homework to be done with input from families or from others not a part of the school environment.

These opening questions are followed by discussion issues that clarify the initial questions and begin to shape the direction of the lesson. Once the stage has been set for the problem, students begin to investigate the situation mathematically. As students work their way through the investigations, it is important that they have the opportunity to share their thinking with others and to discuss their solutions in small groups and with the entire class. Many of the exercises are designed for groups in which each member does one part of the problem and the results are compiled for final analysis and solution. Multiple solutions and solution strategies are also possible, and it is important for students to recognize these situations and to discuss the reasoning behind different approaches. This will provide each student with a wide variety of ways to build his or her own understanding of the mathematics.

In many cases, students are expected to construct their own understanding by thinking about the problem from several perspectives. They do need, however, validation of their thinking and confirmation that they are on the right track, which is why discourse among students, and between students and teacher, is critical. In addition, an important part of the teacher's role is to help students link the ideas within an investigation and to provide an overview of the "big picture" of the mathematics within the investigation. To facilitate this, a review of the mathematics appears in the summary following each investigation.

Each investigation is followed by a Practice and Applications section in which students can revisit ideas presented within the lesson. These exercises may be assigned as homework, given as group work during class, or omitted altogether if students are ready to move ahead.

Student assessments occur after two units in the student book. These can be assigned as long-range take-home tasks, as group-assessment activities, or as in-class work. The assessment pages provide a summary of the lessons up to that point and can serve as a vehicle for students to demonstrate what they know and what they can do with the mathematics. Commenting on the strategies students use to solve a problem can encourage students to apply different strategies. Students also learn to recognize those strategies that enable them to find solutions efficiently. Also included are two student projects.

## Prerequisites

The students should be able to perform basic arithmetic operations with whole numbers, decimals, and fractions. They should understand the concepts of percent and proportion. They should also be able to construct number-line plots, stem-and-leaf plots, box plots, scatter plots, and bar graphs.

## Pacing/Planning Guide

It is not necessary to start on the first section and complete every lesson in sequence. There is a logical progression through the module, but sections and lessons may stand alone. Within a lesson, however, it is important for students not to skip problems. The practice of working odd or even problems will lead to confusion, because concepts will have been missed. Suggested times for lessons are provided.

| LESSON | OBJECTIVES | PACING |
|---|---|---|
| **Unit I: Context and Units** | | |
| Lesson 1: Numbers in Context | Recognize units for numbers in context. | 1 to 2 class periods |
| Lesson 2: Communicating with Data | Recognize how data are used in quality ratings. | 1 class period |
| Lesson 3: Data in Rows and Columns | Make sense of data in tables and other data displays; find patterns and make comparisons in data. | 1 to $1\frac{1}{2}$ class periods |
| Assessment for Unit I | | 1 class period |
| **Unit II: Sorting and Counting** | | |
| Lesson 4: Categorical and Measurement Data | Recognize differences between categorical and measurement data; use appropriate summary techniques for each type of data. | 1 class period |
| Lesson 5: Developing an Investigation Process | Learn to use an investigation approach to solve a problem. | 1 class period |
| Lesson 6: Using Data in the Investigation Process | Use an investigation approach to solve a problem; count, sort, and display data to answer questions. | 1 class period |
| Project: What If Water Cost as Much as Cola? | | 2 class periods |
| **Unit III: Using Percents** | | |
| Lesson 7: Percents and Relative Frequency | Use relative frequency and data-collection techniques to make predictions. | 1 class period |
| Lesson 8: Percents and Visual Comparisons | Use percents to make comparisons; use grids to clarify comparisons. | 1 class period |
| Lesson 9: Percents and Surveys | Interpret the results of a poll using percents. | 1 class period |
| Lesson 10: Percents and Diagrams | Use diagrams to find shooting percents of basketball players; compare percents. | 1 class period |

| LESSON | OBJECTIVES | PACING |
|---|---|---|
| Lesson 11: Percents and Probability | Compare the likelihood of outcomes; state the probability of an event as a ratio or percent; determine profit or loss. | $1\frac{1}{2}$ class periods |
| Assessment for Unit III | | 1 class period |
| **Unit IV: Measurement Data in Experiments** | | |
| Lesson 12: Exploring Centers and Variability | Explore the concepts of center of measures, variability, and bias. | 1 class period |
| Lesson 13: Reaction-Time Experiment | Collect and interpret experimental data to measure reaction time; use measurements in the investigation process. | 1 class period |
| Lesson 14: Burst-Your-Bubble Experiment | Collect experimental data; use measurements in the investigation process. | 1 class period |
| Project: Waiting Time in the Lunch Line | | 2 class periods |
| | | about 4 weeks total time |

## Technology

The amount of technology students can use may vary. Scientific calculators are essential in some lessons. Graphing calculators or computers with spreadsheet and graphing capabilities could be used in some lessons.

## Where to Use the Module in the Curriculum

This module may be used to teach or review important knowledge and skills related to communicating with data. The development of the investigation process helps students to understand how data can be used to answer questions or present convincing arguments. The module may be used in grades 7 and 8, after students have studied percent, as part of a unit on statistics. The activities may also be used in the high-school grades as enrichment or review.

## Use of Teacher Resources

Quizzes, an end-of-module test, solution keys for quizzes and the test, and student activity sheets are available in the back of this teacher's edition. These items are referenced in the *Materials* sections at the beginning of the lesson commentaries.

| LESSON | RESOURCE MATERIALS |
|---|---|
| **Unit I: Context and Units** | |
| Lesson 1: Numbers in Context | |
| Lesson 2: Communicating with Data | |
| Lesson 3: Data in Rows and Columns | *Activity Sheet 1* (Problem 1) |
| Assessment for Unit I | *Unit I Quiz* |
| **Unit II: Sorting and Counting** | |
| Lesson 4: Categorical and Measurement Data | |
| Lesson 5: Developing an Investigation Process | |
| Lesson 6: Using Data in the Investigation Process | *Activity Sheet 2* (Problems 1 and 2) |
| Project: What If Water Cost as Much as Cola? | *Activity Sheets 3 and 4* |
| **Unit III: Using Percents** | |
| Lesson 7: Percents and Relative Frequency | *Activity Sheet 5* (Problems 3 and 5) |
| Lesson 8: Percents and Visual Comparisons | *Activity Sheet 6* (Problems 4–6 and 8)<br>*Activity Sheet 7* (Problems 9 and 10) |
| Lesson 9: Percents and Surveys | |
| Lesson 10: Percents and Diagrams | *Activity Sheet 8* (Problems 4–6 and 9–12)<br>*Activity Sheet 9* (Problems 7 and 8)<br>*Activity Sheet 10* (Problems 13–16) |
| Lesson 11: Percents and Probability | *Activity Sheet 11* (Problems 2–4, 7, and 10) |
| Assessment for Unit III | *Unit III Quiz* |
| **Unit IV: Measurement Data in Experiments** | |
| Lesson 12: Exploring Centers and Variability | *Activity Sheets 12* and *13* (Problem 5) |
| Lesson 13: Reaction-Time Experiment | |
| Lesson 14: Burst-Your-Bubble Experiment | End-of-Module Test |
| Project: Waiting Time in the Lunch Line | *Activity Sheet 14* (Problem 1)<br>*Activity Sheet 15* (Problem 3) |

UNIT I

# Context and Units

## LESSON 1

# Numbers in Context

**Materials:** grid paper
**Technology:** graphing calculator (optional)
**Pacing:** 1 class period or 2 if technology is used and the technology and/or scatter plots are new to students

### Overview

The purpose of this lesson is for students to understand how important context is in determining what numbers represent. The unit attached to a number is essential to the context.

Once students understand the meaning of numbers *in* context, they possess a tool that enables them to recognize reasonable and unreasonable solutions to problems. They can also use numbers to make comparisons.

### Teaching Notes

Some terms introduced in this lesson are *quantified,* that is, counted or measured; and *data,* or numbers used in context. Some discussion may be needed to clarify numbers *in* or *out of context.*

Two examples are:

- Would a score of 95 on a test be good? Is this in or out of context?
- Is a golf score of 95 in or out of context?

Class discussion is important when making the estimates because, in some categories, there may be more than one estimate, and it is important for students to understand that different answers are meaningful and there can be more than one "correct" answer.

Consider reviewing scatter plots. Students should have some experience with scatter plots. If scatter plots are introduced here, more follow-up practice is needed for students to fully understand their value in comparing two variables.

### Follow-Up

Students may want to look up world records for age, speed, and so on. The data in context can be shared in numerical or graphical form.

### Technology

Graphics calculators such as TI-83s could be used to show the scatter plot for Problem 10. This would probably require more time than you may want to spend on this lesson, depending on how much experience the students have had with graphing calculators, as well as how much experience the students have had with scatter plots.

STUDENT PAGE 3

LESSON 1

# Numbers in Context

**Would a score of 95 on a test be good? How about 95 as a golf score? Would you like to score 95 in bowling?**

**Is a 95-page book long? A 95-page homework assignment?**

The number 95 needs a *context* to give it meaning. Almost everything tends to be numbered in one way or another in our society—everything that can be counted, measured, averaged, estimated, or otherwise *quantified.* Calculators and computers help us do amazing things with numbers. But to have a precise meaning in our world, a number must be used in context. Often a number has a unit in its context. Numbers used in context are called ***data.***

OBJECTIVES

Recognize units for numbers in context.

INVESTIGATE

The prices a gas station charges for gasoline are usually written on a sign large enough for drivers to see from the road. This allows customers to compare prices. Numbers such as $112^9$ are often used to indicate price. In this case, the unit is cents per gallon and the price for one gallon of gasoline is 112.9 cents or 1 dollar and 12.9 cents. What other numbers may be displayed for consumers to make decisions about what to buy or where to shop?

## STUDENT PAGE 4

**Discussion and Practice**

**1.** Match each item with an appropriate context.

| Item | Context |
|---|---|
| $9 \times 12$ | test score |
| $6\frac{1}{2}$ | locker combination |
| 21–7 | room size |
| 7–21 | shoe size |
| 102–95 | temperature |
| 95 | population of the United States |
| 75 | time |
| 55 | cost of a car |
| 20,000 | score of a football game |
| 254,000,000 | score of a basketball game |
| 2–30–38 | miles per hour |
| 2:05 | July 21 |

**2.** When is the number 6 a good result? Compile a list of situations or contexts in which 6 is a good result. Be sure to indicate the units in each case.

**3.** When is the number 6 *not* a good result? Compile a list of situations or contexts in which 6 is not a good result. Be sure to indicate the units in each case.

**Practice and Applications**

**4.** Below are two sets of data.

| | |
|---|---|
| \$10.07 | \$12.74 |
| 8.687 gallons | 10.204 gallons |
| Unleaded Regular: $115^9$ | Unleaded Super: $124^9$ |

**a.** Describe the probable setting or context in which you would find these numbers.

**b.** What would be an appropriate unit for $115^9$ and $124^9$ in this context?

**5.** For each unit of data, describe two contexts, one in which 50 is large and one in which 50 is small.

**a.** 50 people **b.** \$50 **c.** 50 mph **d.** 50 minutes

**6.** Ages of young children are often written with months as the unit. Express the age 22 months in two different ways.

## Solution Key

## Discussion and Practice

**1.**

| | |
|---|---|
| test score | 55, 75, 95 |
| locker combination | 2-30-38 |
| room size | $9 \times 12$ |
| shoe size | $6\frac{1}{2}$ |
| temperature | 55,75,95 |
| population of U.S. | 254,000,000 |
| time | 2:05 |
| cost of a car | 20,000 |
| score of a football game | 21–7, 7–21 |
| score of a basketball game | 102–95 |
| miles per hour | 55,75,95 |
| July 21 | 7-21, 21-7 |

**2.** 6 problems correct on a 6-problem quiz, 6 strokes under par in golf, 6 wrong on a 100-point test, 6 points ahead of your opponent in soccer, \$6 per hour for a baby-sitting job, and many others that students may generate

**3.** 6% on a test, 6 strokes over par in golf, 6 right on a 25-question test, 6 points behind your opponent in softball, 6 speeding tickets, 6 suspensions, and many others that students may generate

## Practice and Applications

**4.** Depending on time, this can be assigned to be completed in class or assigned as homework.

**a.** They appear to be the numbers you would see on a gasoline pump when filling your car with gasoline.

**b.** The correct unit for the last numbers in each pair is cents per gallon.

**5.** Possible answers are given.

**a.** 50 people: students in a classroom and fans at a football game

**b.** \$50: price of a concert ticket and money in a savings account

**c.** 50 mph: speed of a car in a parking lot and speed of a car on the freeway

**d.** 50 minutes: amount of time spent doing math homework in an evening and amount of time spent visiting with friends at a party

**6.** $1\frac{10}{12} = 1\frac{5}{6} \approx 1.83$ years

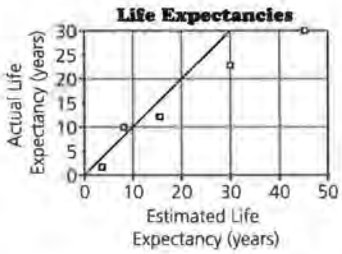
# STUDENT PAGE 5

**7.** Answers will vary.

**8.** Estimates will vary. See table below.

**9.** Possible answer: The life expectancy for the hippomotamus is the greatest.

**10.** Possible answer:




**Note:** If a point lies above the line $y = x$, the actual age is greater than the estimate. If the point lies below the line, the actual age is less than the estimate. By looking at the points' locations with respect to the line $y = x$, students can determine whether their points are over-estimates or under-estimates.

**11.** Answers should focus on the concept of variability. For instance, the life span of the dog is given as a range because there is great variability among types of dogs.

**12.** **a.** 408 months
**b.** 392 weeks
**c.** 18.896 years
**d.** 178.5 months
**e.** 7.08 years

| Animal | Estimate of Life Expectancy | Actual Life Expectancy | Records |
|---|---|---|---|
| **Dog** | | 10–12 years | 24 years |
| **Hamster** | | 2 years | 8 years |
| **Duck** | | 10 years | 15 years |
| **Horse** | | 20–25 years | 50+ years |
| **Hippopotamus** | | 30 years | 40+ years |

---

Use the table below for Problems 7–11.

**Average Life Expectancy for Certain Animals**

| Animal | Estimate of Life Expectancy | Actual Life Expectancy |
|---|---|---|
| Dog | ______ | ______ |
| Hamster | ______ | ______ |
| Duck | ______ | ______ |
| Horse | ______ | ______ |
| Hippopotamus | ______ | ______ |

**7.** Estimate the average longevity, or life expectancy, of each animal listed.

**8.** Use a data source or ask your teacher for information to complete the actual-life-expectancy column in the table.

**9.** Note the variations in the expected longevity among these animals. Which animal typically lives the longest?

**10.** Make a scatter plot with your estimates on the horizontal axis and actual life expectancy on the vertical axis of your graph.

**a.** How do your estimates compare with the actual life expectancy? Explain.

**b.** Compare your estimates with those of another student. Write a sentence that explains how they compare.

**11.** Sometimes life expectancy is given as a range of time. Do you think this is reasonable? Explain.

**12.** Some animals live a very long time. Listed below are some special animals and the length of time each lived. Use the information and guess what the unit of measurement is for the age of each animal.

**Record-Setting Longevity**

| | |
|---|---|
| Joey, K. Ross's canary | Lived to be 408 |
| Fritzy, mouse in Great Britain | Lived to be 392 |
| Flopsy, L. B. Walker's rabbit | Lived to be 18.896 |
| Snowball, Wall's guinea pig | Lived to be 178.5 |
| Rodney, Rodney Mitchell's rat | Lived to be 7.08 |

Source: *Guinness Book of World Records*, 1993

STUDENT PAGE 6

**Extension**

**13.** There is variability in the traits of animals. Not all animals of the same type have the same height, weight, or life expectancy.

Consider this situation: You are a pet-store owner and Buzz Stone comes back to you complaining that the hamster he purchased 12 months ago has died. When Buzz bought the hamster, he asked you about the life expectancy of hamsters and you told him it was about 2 years. What could you tell him?

**Summary**

A number can have many meanings. The meaning depends on the *context* and the *unit*. It is not possible to tell whether a number represents something large or small, or good or bad, without a context. Numbers written in context are ***data***. A range of numbers may be reported when there is *variability* in data. For example, the life expectancy of a horse is 20–25 years.

## Extension

**13.** Possible answer: Discuss with the customer the age of the hamster when it was purchased and the fact that variability in a two-year average age could be a year more or a year less.

LESSON 2

# Communicating with Data

**Materials:** none
**Technology:** none
**Pacing:** 1 class period

## Overview

This lesson introduces a use for numbers that involves more than counting and measuring. Numbers can be used as a tool to rate the quality of things. Products are continually rated and the ratings are reported so that consumers know which products are considered the best. This lesson demonstrates a method using numbers and categories for rating cities. This process could be used for rating products, schools, and so on.

## Teaching Notes

A *Money Magazine* article was used as a resource for this lesson. The rankings are based on data concerning weather, health care, recreation, education, transportation, the arts, crime, housing, and employment opportunities. An explanation of how this ranking was determined is provided in the article.

## Follow-Up

*Consumer Reports* is another excellent resource for student reference to help them learn how information is quantified and products are rated. Students could choose a product they would like to rate, list the data to be collected to determine the ratings, and compare their list to that in a *Consumer Reports* article.

STUDENT PAGE 7

LESSON 2

# Communicating with Data

**What facts about your city make it a great place to live?**

**If you could choose to live in any city in the United States, which one would you choose and why?**

According to *Money Magazine*, July, 1997, the United States' top ten metropolitan areas in which to live are:

1. Nashua, New Hampshire
2. Rochester, Minnesota
3. Monmouth, New Jersey
4. Punta Gorda, Florida
5. Portsmouth, New Hampshire
6. Manchester, New Hampshire
7. Madison, Wisconsin
8. San Jose, California
9. Jacksonville, Florida
10. Fort Walton Beach, Florida

OBJECTIVE

Recognize how data are used in quality ratings.

INVESTIGATE

Decisions about which cities in the Unites States are best in which to live may be made and reported in the media by people who have not even visited those cities. The rankings for the cities above were based on data concerning weather, health care, recreation, education, transportation, the arts, crime, housing, and jobs. What information would you use to rank the top ten metropolitan areas in the United States?

## Solution Key

### Discussion and Practice

1. Aside from those listed in the lesson, students may want to add others like proximity to a beach or mountains, the presence of a favored athletic team, distance from other members of their family, population density, and so on.

2. It often does. For instance, the pay scale for jobs, the cost of an average house, and the perception of a school system's quality influence such decisions.

3. You may want to have students develop a class list of the common features. It may also be helpful to look at a map. Most of the cities are located in New England or Florida.

4. You may want to assign one to each group and have them come up with a list. The lists could be displayed so a class discussion could follow. Sample answers are given.

   **a.** Cars: engine size, environmental concerns, colors available, safety record, reliability record, service record of nearby dealers, mileage

   **b.** Schools: size, availability of special programs, academic record, athletic record, longevity of staff, transportation factors

   **c.** Airlines: on-time record, baggage-handling record, age of planes, friendliness of staff, number of flights

   **d.** baseball stadiums: team appeal, modern or classic design, covered or open, artificial turf or grass, size and seating capacity, location

## STUDENT PAGE 8

**Discussion and Practice**

1. What factors do you think are most important in determining a quality rating for a city?
2. Do you think information regarding how cities are ranked affects choices people make about where they live? Explain.
3. Describe any common features in the cities listed above.
4. Specify information you think would be important to consider in determining a quality rating for each of the following.

   **a.** Cars  **c.** Airlines
   **b.** Schools  **d.** Baseball stadiums

Consider the following question: "How could you rank the major U.S. airlines?" In *USA Today*, April 12, 1994, information was reported for the major airlines that can be used in a quality rating. The table, which looks like an airline report card, includes these four categories.

On-Time Percent: percent of flights that arrive on time
Mishandled Bags: bags that are lost or damaged beyond normal wear
Bumped Passengers: passengers that miss a flight because of overbooking
Average Age of Aircraft: the average age of the planes owned by an airline

**Airline Service Report**

Of the nine major U.S. airlines, only three—Southwest, USAir, and TWA—improved in service last year, the Airline Quality Rating survey says. American, Delta, Northwest, America West and Continental got worse. United held steady. Major airlines are defined as having operational revenue of at least $1 billion.

| On-Time Percent | |
|---|---|
| Southwest | 89.5% |
| Northwest | 85.9% |
| America West | 85.5% |
| USAir | 82.9% |
| TWA | 82.5% |
| American | 80.8% |
| Continental | 79.0% |
| United | 78.4% |
| Delta | 76.7% |

| Mishandled Bags | |
|---|---|
| Southwest | 38.1 |
| America West | 44.1 |
| TWA | 50.5 |
| American | 56.9 |
| Delta | 57.3 |
| Northwest | 58.9 |
| USAir | 59.0 |
| Continental | 61.1 |
| United | 64.8 |

## STUDENT PAGE 9

**Bumped Passengers**

| | |
|---|---|
| American | 0.36 |
| United | 0.36 |
| USAir | 0.68 |
| Delta | 0.73 |
| Northwest | 1.21 |
| TWA | 1.58 |
| Continental | 1.69 |
| America West | 2.10 |
| Southwest | 3.18 |

**Average Age of Aircraft**

| | |
|---|---|
| Southwest | 7.3 years |
| America West | 7.6 years |
| American | 8.9 years |
| Delta | 9.1 years |
| United | 10.8 years |
| USAir | 11.0 years |
| Continental | 15.3 years |
| Northwest | 16.4 years |
| TWA | 18.7 years |

NOTE: Numbers of mishandled bags and bumped passengers are per 10,000 passengers.

Source: U.S. Department of Transportation, *World Aviation Directory*

**5.** Refer to the airline service report above.

**a.** How were the major airlines identified?

**b.** The note below the tables is a key for interpreting some of the numbers. Delta Airlines has a mishandled-bag rating of 57.3. Write a sentence explaining what this number means.

**c.** The number of bumped passengers for United and American Airlines is 0.36 per 10,000 passengers. If one of these airlines serves a million people in a year, about how many of these would be bumped?

**6.** Use the information from the table to identify each of the nine major airlines as *excellent, good,* or *fair.*

**7.** Do you think it is always better for an airline to have newer airplanes? Explain.

**8.** Read the article *Southwest Ranks Best in Airline Survey* on page 10. It was written in the middle 1990s. Does your quality rating system agree with the article's point of view? Explain.

**5. a.** Those having operational revenue of at least $1 billion.

**b.** "57.3 out of 10,000" means that about 57 out of every 10,000 passengers or 573 out of 100,000 passengers will probably have some trouble with the way their bags are handled.

**c.** "0.36 out of 10,000" means that 36 out of every 1,000,000 passengers will be bumped due to overbooking.

**6.** Many answers are possible, as groups may have various overall ratings depending on their rating process. Some groups may simply use the process of adding rating points, while others may give more weight to the categories they may feel are more important, such as whether on-time flights are more/less important than bumped passengers. For each category, students must determine if greater numbers are better than lesser numbers or vice versa.

Probable overall ratings:

Excellent: Southwest, America West, American

Fair: Continental

Good: Northwest, USAir, TWA, United, Delta

**7.** Newer planes may still have some design glitches. Also, older planes may be roomier and therefore more comfortable. Address at what age real problems like metal fatigue can develop. Another issue involves whether or not overhauls actually replace equipment or just repeatedly patch it.

**8.** Before the students complete this exercise, you may want them to write their own "consumer report" that explains the descriptions they assigned to the airlines in Problem 6. They can then compare their report with the information in the article.

## Practice and Applications

9. You may want students to write a "consumer report" on how they identified the "best" car.

# STUDENT PAGE 10

**Southwest Ranks Best in Airline Survey**

Southwest Airlines bumped American in 1993 to become the USA's best major airline.

TWA, worst for three years, left that spot to Continental.

The rankings are based on Department of Transportation statistics. The annual survey—done by Wichita State University and the University of Nebraska—considers such things as lost bags, late flights, age of fleet, complaints, and denied boardings.

Overall, service on the USA's nine major airlines worsened for the fourth year. "Airline travel has become a big bus ride," researcher Dean Headley says. U.S. airlines have lost $11.4 billion the past four years. As their finances improve, Headley expects service to rebound.

American had been No. 1 for three years. A flight attendants' strike in November probably caused more complaints.

Southwest had the fewest lost bags and best on-time performance. That makes sense. Most of its routes are short and fliers tend not to check bags. Southwest also avoids congested airports, which cause delays. American had the fewest denied boardings.

TWA showed the most improvement. Last year, it emerged from Chapter 11 bankruptcy reorganization and became 45% employee-owned.

Source: *USA Today*

**Practice and Applications**

9. The following table lists some of the test data recorded in a rating of some mid-priced sports sedans. Use these data to identify the best and the poorest cars in terms of acceleration, fuel economy, and braking from 60 mph.

**Car-Test Data**

| | Infiniti G20 | Volkswagen Jetta III | Acura Integra | Nissan Altima |
|---|---|---|---|---|
| **Acceleration** | | | | |
| 0–30 mph, sec | 4.0 | 3.6 | 3.6 | 3.3 |
| 0–60 mph, sec | 10.1 | 8.9 | 9.2 | 8.7 |
| 45–65 mph, sec | 6.9 | 5.4 | 5.5 | 5.6 |
| **Fuel Economy** | | | | |
| EPA, mpg (city/hwy) | 24/32 | 18/25 | 25/31 | 24/30 |
| CU's 150-mile trip, mpg | 34 | 27 | 35 | 32 |
| City driving, mpg | 20 | 16 | 21 | 19 |
| Gallons of fuel, 15,000 mi | 515 | 665 | 495 | 650 |
| Cost of fuel, 15,000 mi | $615 | $795 | $595 | $660 |
| **Braking from 60 mph** | | | | |
| Dry pavement, ft | 136 | 134 | 142 | 132 |
| Wet pavement, ft | 149 | 156 | 170 | 149 |

Source: *Consumer Reports*, November, 1994

## Extension

**10.** Answers will vary.

# STUDENT PAGE 11

**Extension**

**10.** Some athletes are models for young people today. In a poll conducted by a Sports Marketing Group, reported in the *Kalamazoo Gazette* on June 23, 1994, the four most important positive character traits of athletes, as identified by 12–17 year olds, were: caring, smart, good sport, and trustworthy. The negative character traits identified by 12–17 year olds were: cheater, cocky, greedy, and weak.

**a.** Do you think these qualities should be used to rank athletes as sports heroes? Explain.

**b.** Would you consider other positive or negative traits? Which ones?

**Summary**

Data on climate, education, health care, recreation, and so on, are used to compare and rank cities. Data are also used to rate or rank airlines and cars. In the development of ranks or ratings, information is quantified and decisions are made about what is favorable and what is unfavorable.

## LESSON 3

# Data in Rows and Columns

**Materials:** *Activity Sheet 1*
**Technology:** none
**Pacing:** 1 to $1\frac{1}{2}$ class periods

### Overview

This lesson introduces students to the organization of data in a table. When the data are organized, comparisons become easier and patterns become more noticeable. The data are also more accessible for manipulations that may need to be performed.

This particular lesson focuses on soda-consumption patterns of people in different regions of the United States. Since soda is part of most students' diet, this lesson provides an opportunity to collect real data from their lives.

### Teaching Notes

*Per capita* may need to be defined for students. For example, 46.7 gallons per-capita consumption means that each person consumes an average of 46.7 gallons of soda per year. Students can calculate their own annual soda consumption by multiplying daily consumption by 365, weekly consumption by 52, or monthly consumption by 12.

### Follow-Up

Students could bring in data tables from newspapers or magazines to discuss. Important features for them to recognize are the title, units, source of data, and the message or information that is conveyed. They may also discuss whether the data could be displayed effectively in a plot or graph.

## Solution Key

### Discussion and Practice

1. Students are encouraged to keep individual records of the soda-consumption estimates for the class. After estimates are recorded for the number of sodas per day, students will need to discuss how soda consumption would be found for the week, month, and year. Discuss the importance of using a standard unit for data collection.

STUDENT PAGE 12

LESSON 3

# Data in Rows and Columns

**What is your favorite kind of soda to drink?**

**Can you describe the advertisements for your favorite soda?**

**Do you think advertisements have affected your preference?**

OBJECTIVES

Make sense of data in tables and other data displays.

Find patterns and make comparisons in data.

Each percent of the soft-drink market is worth about 460 million dollars a year to a company in retail sales. It is, therefore, not surprising that millions of dollars are spent advertising soft drinks. In 1991, per-capita consumption of soft drinks was 46.7 gallons. In this lesson, you will learn about your own soda-consumption patterns and compare them with the patterns of people in many regions of the nation.

INVESTIGATE

About how much soda do you drink in a day, a week, a month, and a year? How could you calculate an estimate for the amount of soda you drink in a year?

**Discussion and Practice**

1. Set up a table like the one below, or use *Activity Sheet 1*. Record individual soda-consumption estimates for each person in the class in terms of numbers of 12-ounce cans.

**Soda Consumption**

| Names | Day | Week | Month | Year |
|---|---|---|---|---|
| | | | | |
| | | | | |
| Averages | | | | |

## STUDENT PAGE 13

2. Refer to your soda-consumption data.
   a. Make a number-line plot of the data for daily soda consumption in 12-ounce cans.
   b. Describe the variability of the class data for daily soda consumption.
   c. Find the average amount of soda consumed each day by students in the class.
3. What is the estimated average amount of soda consumed per student for your class, in gallons per year?
4. Find out how many students attend your school and use that information for the following problems.
   a. Use your class average to estimate the amount of soda consumed daily by students in your school. Be sure to specify the units.
   b. About how much money is spent on soda each day by students in your school?
   c. If all the students in your school were to drink soda from cans, about how many cans would be thrown away each year?

Data are often organized in tables or charts. When data are organized it may be easier to make comparisons, find patterns, recognize variability, and calculate averages. It is also helpful to represent data in plots or graphs.

**Practice and Applications**

The estimates in the table that follows were based on a survey of bottlers. Data include seltzer and club soda marketed by soft-drink bottlers but do not include noncarbonated drinks. *Per capita* means "per person."

2. Students should mention the range of consumption and describe the center of the data. The terms *median, mean,* and *mode* or some appropriate combination of these should be used.

3. Be sure to account for the people who drink no soda at all in some agreed-upon way. Students should decide whether to include the non-soda drinkers or discount their data. Since there are 4 quarts in a gallon, and 32 ounces per quart, there are 128 ounces or $10\frac{2}{3}$ (about 10.67) 12-oz cans per gallon.

4. **a.** Students will need to know the total number of students in the school. One possible option is to discount the total population by a certain percent to account for those students who do not drink soda. This number should match the percent of students in your class who do not drink soda. For instance, if 3 out of 30 students did not drink soda at all, you could reduce the number of students in the school by 10%.

   **b.** You will need to approximate the cost of an average 12-oz can of soda in your area to multiply by your answer in part a.

   **c.** Use the answer in part a and multiply by 365.

## Practice and Applications

**5.** **a.** Answers will vary.

**b.** Answers will vary. For example, in New England 554,500,000 gallons are consumed each year.

**6.** The answers can be obtained from the Total Gallons column. They may also be obtained by multiplying population in millions by per-capita consumption (gallons), with some rounding error. Greatest is the South Atlantic region with $42.8 \times 50.2 \approx 2148.6$ million gallons; least is the Mountain region with $13.4 \times 37.1 \approx 497.1$ million gallons.

**7.** **a.** The greatest average consumption is the one with the greatest per-capita consumption, East South Central; the least average consumption is the one with the least per-capita consumption, Mountain.

**b.** The least per-capita consumption and least total soda consumption are the same, Mountain; but the greatest per-capita consumption, East South Central, and the greatest total soda consumption, South Atlantic, are not the same. This is because the greater population in the South Atlantic region raises the total soda consumption above that in the East South Central region.

**c.** Answers will vary.

**8.** \$43,106,200,000 or about \$43 billion

**9.** Answers will vary, depending on the average class consumption.

## STUDENT PAGE 14

**Yearly Consumption of Soft Drinks in the U.S. by Region**

| Region | Population (millions) | Total Gallons Consumed (millions) | Per-Capita Consumption (gallons) | Retail Sales (millions of dollars) |
|---|---|---|---|---|
| New England | 12.8 | 554.5 | 43.3 | $ 2,076.9 |
| Middle Atlantic | 37.6 | 1,737.9 | 46.2 | $ 6,936.5 |
| East North Central | 42.1 | 2,033.3 | 48.3 | $ 7,518.0 |
| West North Central | 17.8 | 868.4 | 48.8 | $ 2,941.8 |
| South Atlantic | 42.8 | 2,152.8 | 50.2 | $ 7,181.0 |
| East South Central | 15.4 | 845.6 | 54.9 | $ 2,862.5 |
| West South Central | 27.1 | 1,303.8 | 48.1 | $ 4,570.5 |
| Mountain | 13.4 | 496.8 | 37.1 | $ 2,013.2 |
| Pacific | 37.6 | 1,538.2 | 40.9 | $ 6,268.8 |
| Totals | 246.8 | 11,531.2 | 46.7 | $43,106.2 |

Source: *Beverage World*, May, 1990

**5.** Refer to the table above.

**a.** In which region do you live?

**b.** How much soda is consumed each year in the region where you live? Write your answer without a decimal point and with appropriate units.

**6.** In which two regions are the greatest number of gallons of soda consumed? The least?

**7.** Refer to the per-capita column in the table above.

**a.** Identify the regions in which soda consumption is the greatest and the least.

**b.** Are these the same regions you identified in Problem 6? Explain why or why not.

**c.** What do you observe about the consumption of soda in the U.S.?

**8.** How much money in dollars is spent on soda each year in the U.S.?

**9.** How does the U.S. per-capita total compare to the average amount consumed by students in your class in gallons per year?

## Extension

**10.** To find the per-capita values, the total number of gallons is divided by the population. It is important for students to understand the relationship between the numbers in these two columns, which are both given in millions.

**11.** 46.7 is the total number of gallons divided by the total population, taking into account the fact that the regions have different population sizes. The number 46.4 is a straight average of the per-capita column. If all regions had the same population, this would be the correct answer. Students should understand how regional population affects per-capita consumption.

**12.** The prices differ for sodas depending on the regional area. For example, in New England the average cost of a gallon is $3.75 (2076.9 retail dollars divided by 554.5 total gallons). In the South Atlantic region, a similar calculation yields about $3.34 per gallon.

## STUDENT PAGE 15

**Extension**

**10.** Explain how the data in the per-capita column are calculated.

**11.** If you average the data in the per-capita column, will you get 46.4 instead of 46.7? Explain why 46.7 is really the correct answer.

**12.** Are soft-drink prices the same in different regions of the country? Justify your answer.

**Summary**

When data are organized in tables, it is usually easier to make comparisons and find patterns. The arrangement of data in tables facilitates the calculation of averages.

**Summary for Unit I**

- A number can have many meanings. The meaning depends upon the *context* and the *unit*. It is not possible to tell whether a number represents something large or small, good or bad, without a context. Numbers written in context are called ***data***. A range of numbers may be reported when there is *variability* in data. For example, the life expectancy of a cat is 10–12 years.
- In the development of ranks or ratings, information is quantified and decisions are made about what is favorable and what is unfavorable. Quantified information may include counts such as number of people, averages of several numbers, percents, and rates such as miles per gallon.
- Data are often organized in tables. When data are organized it may be easier to make comparisons, find patterns, recognize variability, and calculate averages. It is also helpful to represent data in plots or graphs.

# Assessment for Unit I

**Materials:** none
**Technology:** none
**Pacing:** 1 class period

## Solution Key

1. Possible answers: 10 on a 10-point quiz, 10 pins knocked down in bowling
2. Possible answers: price, size of screen, quality of picture, remote control, audio features, alarm and sleep timer, channel blockout, appearance

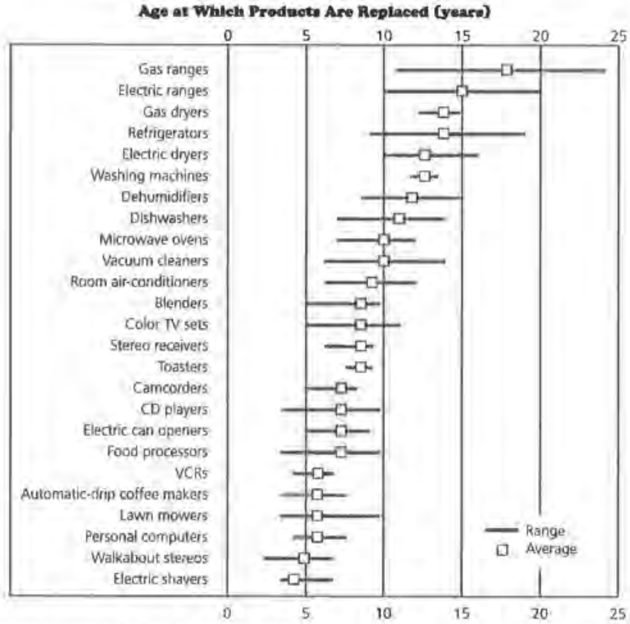
STUDENT PAGE 16

ASSESSMENT

# Assessment for Unit I

1. List two situations for which 10 is a good result. Specify the units in each case.
2. Specify information you think would be important to consider in determining a quality rating for color televisions.
3. The following chart shows how long things last or the age at which products are replaced in years. Use the data for parts a–f on page 17.

**Age at Which Products Are Replaced (years)**




Source: *Appliance Magazine*; a Dana Chase publication

3. **a.** Microwave ovens, vacuum cleaners

   **b.** Between 5 and 11 years; about 8 years

   **c.** Electric shavers, walkabout stereos

   **d.** Gas ranges; they last from 11 to 24 years; the range is 13 years.

   **e.** Toasters have the shortest range (2 years); they last about 8 years, so you would expect to buy two toasters in 15 years.

   **f.** Some products last fewer years than the average, and others last longer; notice that the average is not always in the middle of the range.

4. Possible answers:

   In San Francisco, the temperatures are not extremely hot or cold. Normal daily temperatures range from the low 40s to the high 60s in degrees Fahrenheit. It does not snow and there are no thunderstorms. It is very rare for temperatures to go below freezing or above 90 degrees. Each winter month there are about 10 days with measurable precipitation and clouds. Possible sunshine is in the 60%–65% range. If you were planning a trip to San Francisco, you should take a jacket or sweater for cool weather.

   It is often cloudy at Mt. Rainier National Park. Temperatures usually range from the low teens in winter to the low 70s in degrees Fahrenheit in summer. Snow is possible even in the summer months, and it can snow as many as 150 inches each month in the winter. There are more cloudy days than clear days, especially in the winter. Thunderstorms can occur in the summer months. If you are planning to visit Mt. Rainier, be prepared for cold weather.

## STUDENT PAGE 17

**a.** Which products have to be replaced about every 10 years?

**b.** About how often does a color television have to be replaced?

**c.** Which products have to be replaced most often?

**d.** Which product has the greatest range of years for replacement time? What is the range?

**e.** About how many new toasters should someone expect to buy in a period of 15 years?

**f.** Explain why the age a product has to be replaced is given as a range.

**4.** The following tables describe the weather conditions in Mt. Rainier National Park, about 100 miles from Seattle, Washington, and the Golden Gate National Recreational Area in San Francisco, California. After you study the tables of numbers, write a paragraph for each location describing the weather and what you might expect if you planned to visit these areas.

**Mount Rainier National Park**

| Weather Parameters | Month | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | J | F | M | A | M | J | J | A | S | O | N | D |
| **Temperature** | | | | | | | | | | | | |
| Normal Daily Maximum | 34 | 38 | 43 | 51 | 59 | 64 | 74 | 75 | 68 | 57 | 42 | 34 |
| Normal Daily Minimum | 12 | 15 | 19 | 25 | 32 | 37 | 42 | 41 | 35 | 29 | 22 | 19 |
| Extreme High | 55 | 60 | 73 | 80 | 88 | 93 | 101 | 97 | 94 | 85 | 65 | 61 |
| Extreme Low | −38 | −35 | −18 | −3 | 13 | 22 | 27 | 24 | 18 | −5 | −12 | −16 |
| Days Above 90° | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 1 | 0 | 0 | 0 |
| Days Below 32° | 30 | 28 | 30 | 27 | 17 | 5 | 1 | 1 | 9 | 21 | 28 | 30 |
| **Precipitation** | | | | | | | | | | | | |
| Normal | 14.6 | 10.2 | 8.9 | 6.3 | 4.0 | 3.8 | 1.6 | 2.9 | 4.6 | 7.7 | 12.1 | 15.9 |
| Maximum | 30.4 | 20.8 | 19.5 | 12.5 | 9.1 | 8.0 | 6.0 | 7.2 | 15.2 | 23.6 | 25.4 | 29.1 |
| Maximum 24-Hr Precipitation | 5.7 | 4.8 | 3.3 | 5.4 | 2.2 | 2.8 | 2.3 | 2.5 | 4.5 | 5.3 | 7.9 | 7.9 |
| Maximum Snowfall | 193 | 182 | 155 | 107 | 46 | 9 | 6 | T | 27 | 65 | 138 | 164 |
| Days with Measurable Precip. | 22 | 20 | 21 | 19 | 16 | 15 | 9 | 11 | 13 | 16 | 20 | 23 |
| Average No. Thunderstorms | 0 | 0 | 0 | 0 | 1 | 2 | 1 | 2 | 1 | 0 | 0 | 0 |
| **Sunshine/Cloudiness** | | | | | | | | | | | | |
| No. Clear Days | 3 | 2 | 3 | 3 | 4 | 6 | 13 | 12 | 10 | 6 | 3 | 2 |
| No. Partly Cloudy Days | 2 | 3 | 4 | 5 | 6 | 5 | 7 | 8 | 6 | 5 | 3 | 2 |
| No. Cloudy Days | 26 | 23 | 24 | 22 | 21 | 19 | 11 | 11 | 14 | 20 | 24 | 27 |
| % Possible Sunshine | 28 | 35 | 42 | 50 | 52 | 55 | 65 | 65 | 55 | 45 | 30 | 20 |

# STUDENT PAGE 18

**Golden Gate National Recreation Area**

| Weather Parameters | Month | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | J | F | M | A | M | J | J | A | S | O | N | D |
| **Temperature** | | | | | | | | | | | | |
| Normal Daily Maximum | 56 | 59 | 59 | 60 | 61 | 63 | 64 | 65 | 67 | 66 | 63 | 57 |
| Normal Daily Minimum | 43 | 44 | 44 | 44 | 47 | 50 | 51 | 52 | 52 | 49 | 46 | 44 |
| Extreme High | 74 | 77 | 79 | 84 | 90 | 83 | 81 | 94 | 91 | 91 | 87 | 79 |
| Extreme Low | 29 | 30 | 32 | 32 | 37 | 39 | 42 | 41 | 40 | 35 | 30 | 27 |
| Days Above 90° | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Days Below 32° | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **Precipitation** | | | | | | | | | | | | |
| Normal | 5.3 | 3.7 | 3.5 | 2.1 | 0.6 | 0.2 | 0.1 | 0.2 | 0.4 | 1.6 | 3.0 | 4.4 |
| Maximum | 11.4 | 10.8 | 9.4 | 7.4 | 4.1 | 1.4 | 1.0 | 1.6 | 3.7 | 11.0 | 9.5 | 13.8 |
| Maximum 24-Hr Precipitation | 4.9 | 2.4 | 2.0 | 2.3 | 1.1 | 1.1 | 0.6 | 0.6 | 2.6 | 4.9 | 2.4 | 4.1 |
| Maximum Snowfall | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Days with Measurable Precip. | 11 | 10 | 10 | 6 | 3 | 1 | 1 | 1 | 2 | 4 | 8 | 10 |
| Average No. Thunderstorms | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **Sunshine/Cloudiness** | | | | | | | | | | | | |
| No. Clear Days | 9 | 8 | 10 | 11 | 13 | 15 | 17 | 15 | 16 | 14 | 11 | 9 |
| No. Partly Cloudy Days | 7 | 7 | 9 | 10 | 11 | 10 | 11 | 12 | 10 | 9 | 8 | 8 |
| No. Cloudy Days | 15 | 13 | 12 | 9 | 7 | 5 | 3 | 4 | 4 | 8 | 11 | 14 |
| % Possible Sunshine | 56 | 62 | 69 | 73 | 72 | 73 | 66 | 65 | 72 | 70 | 62 | 53 |

Source: *The Complete Guide to America's National Parks*, 1990–1991

UNIT II

# Sorting and Counting

LESSON 4

# Categorical and Measurement Data

**Materials:** none
**Technology:** none
**Pacing:** 1 class period

## Overview

In Lesson 2, students were informally introduced to categorical data. This lesson formally introduces the idea that there are different types of data—data classified in categories, and data that can be measured. Manipulation of data differs depending on the kind of data collected; therefore, it is important that students understand these classifications. They may think it is appropriate to average any set of numbers, but they need to recognize that there are different kinds of data and, in some cases, averages do not make sense.

## Teaching Notes

Emphasize that measurement data can take on any value on the real-number line. It is not appropriate to say that measurement data are numbers and categorical data are words. It is possible for numbers to be used as categorical data. For example, movies are ranked 1, 2, 3, or 4 stars. These numbers are categories. Some terms may need to be discussed, depending on the exposure students have had with them. Some of these terms may include:

- *counts, modal categories*
- *mean, median,* and *range*
- *quartiles, number-line plots, plots over time, box plots, scatter plots, histograms,* and *stem-and-leaf plots*

## Follow-Up

Students could look for examples of categorical and measurement data in the newspaper or other publications.

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LESSON 4

# Categorical and Measurement Data

**How are movies rated for different audiences?**

**How much money does it cost to make a movie?**

You have been using data that are numbers in context. Data come in many forms. Some data are not numbers at all. Some data, called *categorical* data, only identify a category for an object or observation. For example, there are two categories for gender, male and female. Some categories for movie ratings are G, PG, PG-13, and R.

For some other data, a numerical value can take on any value on the real-number line. This is called *measurement* data. Measurement data use appropriate units to measure an observation or outcome. The time in minutes it takes you to travel to school is an example of measurement data. The amount of money required to make a movie is measurement data.

**OBJECTIVES**

Recognize differences between categorical and measurement data.

Use appropriate summary techniques for each type of data.

**INVESTIGATE**

Sometimes categorical data can be ordered or ranked. A restaurant rating of Excellent, Good, Fair, or Poor is an example of ordered categories.

Measurement data have numerical meaning, and it may make sense to add them, subtract them, and find averages. These data are usually summarized with a *measure of center* and a *measure of spread*. The measure of spread indicates variability.

Categorical data can be summarized by:

Counts
Percents
Modal categories
Bar graphs

## Solution Key

### Discussion and Practice

**1.** Possible answers are given.

**a.** Ice-cream flavors: chocolate, vanilla, strawberry

**b.** Rank favorite flavors 1, 2, 3, . . . . Then add the ranks or choose the flavor with the greatest number of highest ratings.

### Practice and Applications

**2.** **a.** Answers will vary.

**b.** A mean or median wake-up time could be calculated. A range of times could also be used. You might want to discuss with the class which they feel best gives the true picture about when they get up. A line plot or box plot could give a feel for the spread of the data and how times are clustered.

**c.** Answers will vary.

**d.** Answers will vary.

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Measurement data can be summarized by:

Median
Mean
Range
Quartiles
Number-line plots
Plots over time
Box plots
Scatter plots
Histograms
Stem-and-leaf plots

**Discussion and Practice**

**1.** Each heading below can be used to describe a data set. Choose at least five headings to discuss. For each one:

**a.** Identify at least three items that could be in the data set.

**b.** Indicate what method you would prefer to use to summarize each data set.

| | |
|---|---|
| Ice cream flavors | Automobile colors |
| Gender | Race |
| Dominant hand | Citizenship |
| Last names | Letter grades |
| Movie ratings | Temperature |
| Age | Weight |
| Height | Blood pressure |
| Distance | Family size |
| Travel time | Grade-point average |
| Colors of shirts worn today | Elevation |

**Practice and Applications**

**2.** Collect data to answer each question and write an appropriate data summary.

**a.** What are the first three digits of the telephone numbers of students in your class?

**b.** What time do students in your class get up in the morning on a school day?

**c.** What was the last movie seen at a movie theater by students in your class?

**d.** Which television station is watched most often on a given day?

## STUDENT PAGE 23

**3.** Possible answers are given.

**a.** How many children live in your household? (measurement)

**b.** How old are you? (measurement)

**c.** What color are your eyes? (categorical)

**d.** What is your favorite subject in school? (categorical)

**e.** Where would you go on vacation if you could go anywhere? (categorical)

**4.** **a.** Good choices include mean, median, and range. A stem-and-leaf plot that gives an indication of the concentration of ages would be helpful. A box plot would also be appropriate. In a hospital, it would be important to know that many of the patients were elderly.

**b.** Mean, median, and range could be calculated since this is measurement data. A box plot would show the range of temperatures and quartiles.

**c.** Putting the numbers into the categories mentioned—probably in order—provides an idea of the types of players available for the team. Averaging is not useful, but counts and percents are reasonable. A stem-and-leaf plot is reasonable.

**3.** List five questions you could ask in a survey that require different kinds of data.

In order to work with data using a calculator or computer, we often use numbers to represent all variables, even some variables that are categorical. For example, gender is sometimes coded 1 for male and 2 for female; and letter codes are sometimes given values like 4 for A, 3 for B, 2 for C, and so on. In other instances, data that were originally measurement data may be grouped as categories. For example, a category called "twenty-somethings" may be used to collect ages such as 23, 27, 29, and so on. "Teens" describes those from ages 13 to 19.

**4.** Consider the following numbers:

5, 23, 34, 51, 52, 66, 67, 73, 74, 81, 82

Suppose all of these numbers are in use in each of the situations below. Summarize the data in appropriate ways.

**a.** The numbers represent ages of hospital patients on a certain day.

**b.** The numbers represent daily temperatures on a sample of 11 days during the year.

**c.** The numbers represent football-jersey numbers for the Pittsburgh Steelers. Hint: 1–19 are used for quarterbacks only; 20–39 are used for running backs; 50–59 are used for centers; 60–69 are used for guards; 70–79 are used for tackles; and 80–89 are used for ends.

### Summary

Different types of data values are appropriate for different problems. Categorical data represent the number of objects or observations that fall in clearly defined categories. Measurement data can be represented by any values on the real number line. Data may be sorted, counted, ordered, and summarized.

## LESSON 5

# Developing an Investigation Process

**Materials:** none
**Technology:** none
**Pacing:** 1 class period

This lesson continues to allow students to practice collecting and organizing data sets to investigate a problem or answer a question. This particular lesson focuses on data collection of dates and introduces the usefulness of ordering, grouping, and plotting data. The purpose of the lesson is to develop a process for answering the question: Are certain months or dates more popular for birthdays than others? This process will be used to answer other questions in future lessons.

### Teaching Notes

The days numbered 1–12 could be confused with the months if it were not clear whether the month or the day came first. When students collect data in this lesson, they must know if it is in the month/day or day/month form. Emphasize that when data are collected on a written survey, care must be taken to avoid such confusion.

Problems 1–5 present a class-based activity, since class data need to be collected from each individual, and then compiled and organized. Before the data are collected, it may be interesting for students to discuss which months they think will be the most popular birthday months and why. Emphasize that a thoughtful answer to a question is important, and a thoughtful answer supported by data is very convincing.

Students are expected to know how to create number-line plots, bar graphs, and stem-and-leaf plots.

### Technology

Technology is not needed in this lesson, but it could be introduced to show programs that allow people to figure out which days of the week they were born.

### Follow-Up

It is important for students to understand the investigation process. It would be interesting to compare the class data with birthday data on another population such as birthdays of presidents or birthdays of popular singers.

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LESSON 5

# Developing an Investigation Process

**Do any students in your class share the same birthday?**

**What do you think is the most popular month for birthdays?**

OBJECTIVE

Learn to use an investigation approach to solve a problem.

INVESTIGATE

Birthdays of famous people are often reported in the newspaper or on the radio; they may also be celebrated as holidays. For instance, Dr. Martin Luther King was born January 15, 1929. This information may be written using numbers, 1-15-29. The context of the problem is important when using such data shortcuts. Only the fact that the discussion is about birthdays makes it clear that 1-15-29 is not a locker combination.

In many countries around the world, dates are written with the day of the month first.

Emily was born September 18, 1978. In some countries, this is written symbolically as 18-9-78. How would you write your birth date using this symbolism?

On which days of the year would you be confused about the date if you were not certain whether the month or day came first? How would you write twenty years from today's date in symbols?

**Discussion and Practice**

Birthdays are special days. Is today the birthday of someone in your class? Is someone having a birthday this week? This month? How many people have birthdays in the summer?

Many problems can be approached by collecting data and analyzing the results. Below is an outline of an approach that you will use in solving such problems.

## Solution Key

### Discussion and Practice

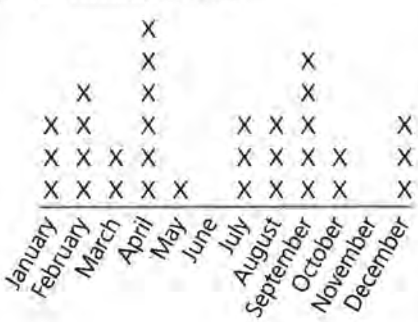
1. A data set is collected. The data set will be specific to individual classes; it is the information that can be used to answer the question.

2. To answer this question a number-line plot is a great help, giving the class a clear picture of the spread of birthdays throughout the month. Simply label a number line from 1 to 31 and put an X above the number representing each student's birthday.

3. Most students do not remember the day of the week they were born, and without the aid of a computer program or other device that information is not available to the class.

4. The number-line plot shows how many students were born on the first day of the month. To determine how many students were born on the last day of the month, the month for each student must be known.

5. **a.** Possible answer:




STUDENT PAGE 25

**Investigation Process**

*Problem:* Identify the problem, usually by stating a question that needs to be answered.

*Data Collection:* Collect data needed to answer the question.

*Data Presentation:* Use tables, graphs, and charts to give a visual summary of your data. Also calculate any appropriate numerical summaries such as measures of center or spread.

*Conclusion:* Write a paragraph describing your conclusions about the answer to the question. You may also highlight things that your graph shows to be true, such as the meanings of clusters and gaps, the center and spread of the data, and so on.

The problem for investigation in this lesson is this: **Are certain months or dates more "popular" for birthdays than others?**

1. As a class, record every person's birthday. This is the *data set* that will be needed to complete the following investigations. As you will see, this data set will be used to find the solution to the problem.
2. Use your data to answer the following questions.
   a. On which days of the month were students born?
   b. Were more students born on even days or odd days?
   c. Which day of the month is the most common for birthdays?
3. Can your data be used to determine which days of the week students were born? Explain.
4. Can these data be used to determine how many students were born on the first day of the month? The last day of the month? Explain.
5. It is often helpful to create plots to identify patterns in data.
   a. Plot the frequency for each month in the data set.
   b. Plot days of the month students were born using a stem-and-leaf plot.
   c. Summarize each of the above plots in words.

**b.** Possible answer:

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 1 | 12 | 27 | | | |
| 2 | 3 | 5 | 21 | 25 | | |
| 3 | 14 | 17 | | | | |
| 4 | 1 | 3 | 6 | 9 | 10 | 20 |
| 5 | 7 | | | | | |
| 6 | | | | | | |
| 7 | 2 | 23 | 30 | | | |
| 8 | 6 | 13 | 30 | | | |
| 9 | 11 | 14 | 29 | 29 | 30 | |
| 10 | 14 | 28 | | | | |
| 11 | | | | | | |
| 12 | 6 | 25 | 28 | | | |

**c.** The summaries can be simple statements about the way the data are distributed.

## Practice and Applications

**6–8.** Answers will vary. Discuss the different clusters.

**9.** Encourage students to make observations or conjectures based on the data collected. Data from several classes may be compared or compiled in one graph.

STUDENT PAGE 26

**Practice and Applications**

When you presented your data, you probably arranged the months or days in order. When data can be ordered, it is often helpful to do so. A reasonable pattern for ordering birthday data is to arrange the months in order from January to December and the days in increasing order for each month.

Data that can be ordered may also be grouped. Group your data to answer the following.

**6.** How many students were born in the summer?

**7.** How many students were born in January or February?

**8.** How many students were born on or before the 15th day of any month?

**9.** Write a short paragraph that summarizes your findings based on any patterns you noticed in the collected data.

**Summary**

Data are often used to answer questions and support arguments. The investigation process outlined in this lesson provides a structure for using data in problem solving. It includes:

- Ask a question that identifies a problem.
- Collect data to answer the question.
- Study the data for patterns. Create appropriate tables, graphs, and plots. Graphs and plots may include number-line plots, box plots, stem-and-leaf plots, histograms, scatter plots, and bar graphs.
- Write a summary that describes your conclusions about the answer to the question.

## LESSON 6

# Using Data in the Investigation Process

**Materials:** *Activity Sheet 2,* small packages of M&Ms
**Technology:** graphing calculator (optional)
**Pacing:** 1 class period

This lesson provides practice using the investigation process to collect and organize data to answer a question. A number of problem-solving strategies are used. By estimating, students can focus on the process and check for reasonable solutions. The concept of variation from the mean is introduced in Problem 5.

### Teaching Notes

Students will need to work in groups as well as be able to interact as a whole class. Data will eventually be compiled for the whole class. Students need to be reminded that the M&Ms need to be used as a manipulative and may be eaten only after all data is recorded; otherwise, the data may be skewed. It may be helpful for students to think about an average package, how the average package was determined, variability from the average package, and how that variability was measured (closeness score). Data from this lesson should be saved to use in Lesson 7.

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LESSON 6

# Using Data in the Investigation Process

**Have you ever wondered how many M&Ms there are in a small package?**

**Do certain colors appear more frequently?**

**Which colors do you prefer?**

**When you eat the M&Ms, do you group them by color?**

In this lesson, you will use the investigation approach you learned in the previous lesson to look at packages of M&Ms and solve a problem.

OBJECTIVES

Use an investigation approach to solve a problem.

Count, sort, and display data to answer questions.

INVESTIGATE

Based on previous experiences of eating M&Ms, would you expect the total number of M&Ms in a small package to vary much? Would you expect the number of one color to vary by very much?

STUDENT PAGE 28

## Solution Key

## Discussion and Practice

1. Be sure the students make their color predictions *before* they open the packages of M&Ms. To get information for the conclusion, group counts should be collected and averaged for each color.

2. a. Answers will vary.

   b. Answers will vary. Discussion may be needed on how to find the class mean for the frequency of each color.

   c. Answers will vary. The class will need to decide whether to allow negative differences, or whether the differences are going to be limited to the absolute values. The total of differences should be calculated only with absolute values.

**Discussion and Practice**

Work together in a small group. Copy the table below or use *Activity Sheet 2*. Each group should have a small package of M&Ms.

**M&Ms Color Data**

| Color | Individual Estimate of Frequency | Actual Frequency | Class Mean Frequency | Difference (Actual Frequency minus Class Mean Frequency) |
|---|---|---|---|---|
| ______ | ______ | ______ | ______ | ______ |
| ______ | ______ | ______ | ______ | ______ |
| ______ | ______ | ______ | ______ | ______ |
| ______ | ______ | ______ | ______ | ______ |
| ______ | ______ | ______ | ______ | ______ |
| ______ | ______ | ______ | ______ | ______ |
| Totals | ______ | ______ | ______ | ______ |

Closeness score for actual counts ______

Closeness score for estimates ______

***Problem:* How many M&Ms of each color would you expect to find in a small package? Specifically, how many green M&Ms would you expect?**

1. Before you open your package of M&Ms, complete the "Individual Estimate of Frequency" column in your table by guessing how many of each color of M&Ms and the total number of M&Ms are in your package.

2. *Data Collection*

   a. Sort and count the number of M&Ms for each color and record the information in the "Actual Frequency" column.

   b. Share the information with your classmates. Find the class mean, or average, for the frequency of each color and complete the "Class Mean Frequency" column.

   c. Complete the "Difference (Actual Frequency minus Class Mean Frequency)" column.

## STUDENT PAGE 29

3. *Data Presentation*

   Create an appropriate plot for the data.

4. *Conclusion*

   Write a paragraph describing how many M&Ms of each color you would expect to find in a package. This will be referred to as the "average package" for the class.

5. Find out how close your counts are to the average counts.

   a. Use the difference column on your table to figure out a number that indicates a closeness score for your data. A closeness score should summarize the differences between your package and the average package. Identify the package closest to the class average.

   b. Create a graphical presentation for the closeness of your color counts to the class averages.

   c. How many small packages would you expect to open to collect a total of at least 25 green M&Ms? Will this number of packages always give you 25 green M&Ms? Explain.

6. In Problem 5, you used a closeness score to compare packages of M&Ms with the average package. Use the same closeness score procedure to determine which group in the class had estimates closest to the average package.

**Practice and Applications**

7. Now try another investigation involving your package of M&Ms. The problem is: About how much does one M&M weigh?

   a. Estimate how much one M&M weighs.

   b. Use the weight and number of M&Ms in an entire package to find an average weight of one M&M for each package. Share the information with other groups. Record the information for each package.

   c. Create a plot to show the variability in the estimated weights from one package to another.

   d. Write a summary of the results.

**3.** Answers will vary, but the plots should summarize the class data.

**4.** Answers will vary, but paragraphs should include discussion of the "average package" for the class. The average package should include the individual averages of each of the colors for the M&Ms.

**5.** **a.** If absolute values are used, students may simply add the differences. Another possibility is to average the absolute values. Students may have another method that you feel works as well. If negative numbers are used, adding the differences may be misleading; the sum can equal zero when the differences are actually quite large.

**b.** Possibilities include bar graphs with the difference stacked on top and arrows showing whether the difference is positive or negative; a line with dots above and below the line that represent the average for each color. Accept any reasonable representations.

**c.** By now, students have an average number of green M&Ms per bag. The quotient of 25 divided by this number will give an estimate of the number of bags required to get 25 green M&Ms. This number will not always give you 25 green M&Ms. Not all packages contain the average number of green M&Ms.

**6.** Answers will vary. You may need to ask each group to use the same closeness procedure outlined above in order to be able to find the group whose estimate was best.

## Practice and Applications

**7.** **a.** Answers will vary.

**b.** Answers will vary, but a ratio is a reasonable representation: total weight/number of M&Ms.

**c.** Answers will vary. Since M&Ms are sold by weight and are fairly uniform in size, there may not be much variability. A stem-and-leaf plot displays the information well.

**d.** Answers will vary, but should include the average weight and a statement about variability.

# STUDENT PAGE 30

**Summary**

The investigation process includes:

- Identification of the problem
- Data collection
- Data presentation
- A summary of results

The summary may include information about the center of the data and how close the data are to the center. A measure of closeness of the data to the center indicates the amount of variability in the data.

**Summary for Unit II**

- Different types of data values are appropriate for different problems. *Categorical* data represent the number of objects or observations that fall in clearly defined categories. *Measurement* data can be represented by any values on the real-number line. Data may be sorted, counted, ordered, and summarized.
- Data are often used to answer questions and support arguments. The investigation process outlined in this section provides a structure for using data in problem solving. The investigation process includes:
  - Identification of the problem
  - Data collection
  - Data presentation
  - A summary of results
- Appropriate plots may include number-line plots, box plots, stem-and-leaf plots, histograms, scatter plots, and bar graphs. The summary may include information about the center of the data and how close the data are to the center. A measure of closeness of the data to the center indicates the amount of variability in the data.

PROJECT

# What If Water Cost as Much as Cola?

**Materials:** *Activity Sheets 3 and 4,* dimensions of classroom; 1-gallon container, measurable rectangular container, ruler, water, tape measure (optional for Problem 5)
**Technology:** graphing calculator with list capabilities or spreadsheet program (optional)
**Pacing:** 1 class period for presentation; 2–5 days of data collection (outside of class); 1 class period for follow-up of data collection, and group time to answer questions

## Overview

This project is designed as a real application of the previous lessons in this unit. Data collection and organization are essential components of the project. Using the investigation process, students interpret the results of their data and create visual representations in the form of plots and graphs. This project reinforces a growing awareness about the need to conserve water, a natural resource. In some parts of the country, water use is restricted at certain times of the year.

## Teaching Notes

A class discussion of students' estimates of their daily water usage should provide a lively introduction to this project. The estimates for a gallon of soda from Lesson 3 ranged from \$3.34 to \$4.07 per gallon, with a national average of \$3.74. The class needs to make a decision about what number they will use in this project. You could also use different everyday liquids instead of soda, such as gasoline or milk. A reasonable estimate for the cost of water is \$2 for 1,000 gallons.

This should be assigned as a several-day project. Data should be collected on *Activity Sheet 3*. Students should record their data on a new activity sheet each day to observe patterns or trends. Keep the number of days low for the data collection, because the students are more likely to record their water usage accurately if the collection time is not too long. One week may be *too* long. The questions are to be answered individually or in groups after each student has collected the data.

## Follow-Up

The students may also use their water-consumption amounts to determine how much they would spend for water if water cost as much per gallon as milk or gasoline.

## Technology

Technology could be used in this unit. If students are familiar with spreadsheets, much of the data could be put into spreadsheets making some of the calculations less cumbersome. If spreadsheets are used, computers will be needed, or possibly even a graphing calculator with statistical lists.

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PROJECT

# What If Water Cost as Much as Cola?

**Has water use ever been limited in your city?**

**What steps can you take to conserve water?**

**When you use water, do you think about how much it costs?**

Gasoline and fuel oil were once thought so plentiful and inexpensive that people did little to conserve them. Times have changed, however, and today people realize that gasoline and oil are limited resources and should be used efficiently. In certain parts of the country today, water is still regarded as plentiful. In those areas, few people adjust their daily activities to conserve water.

INVESTIGATE

About how much water do you think you use in a day? About how much do you think each gallon of water costs?

**Discussion and Practice**

For a period of five consecutive days, each time you perform one of the activities listed on *Activity Sheet 3*, record a mark in the "Tally" column. Remember to include tally marks for activities that were performed for you such as washing clothes. If there are four people in your family and approximately $\frac{1}{4}$ of the clothes in the washing machine are yours, you should record 7.5 gallons for each use. Then complete the "Sum of Tallies" column. The totals should approximate the amount of water you used for those days.

## Solution Key

## Discussion and Practice

1.

| | |
|---|---|
| Taking a bath | 30 gallons |
| Washing clothes | 30 gallons |
| Shower | 18 gallons |
| Washing dishes in an automatic dishwasher | 11 gallons |
| Washing dishes by hand | 10 gallons |
| Flushing toilet | 4 gallons |
| Cooking and drinking | 3 gallons |
| Brushing teeth (water running) | 2 gallons |
| Washing hands | 1 gallon |

2. Answers will vary.
3. Answers will vary.
4. Answers will vary. Students may want to figure a daily total amount of water first, then use the daily total and multiply it by 7 for a week, by 30 for a month, or by 365 for a year.
5. **a.** About 7.5 gallons; to help students visualize this, fill a measurable rectangular container with water and then calculate from the amount of water and the volume of the container the amount of water in a cubic foot.

   **b.** Answers will vary.
6. Answers will vary. Students might calculate the amount by changing $2.54 per 1,000 gallons to $0.00254 per gallon and multiplying by the number of gallons or by dividing the number of gallons first by 1,000 and then multiplying by $2.54.

   Involved in the family calculation, of course, is the assumption that all members use about the same amount of water. A student could reason to a different, nonequal answer, but would need to be very clear why that way is better. Monthly and yearly totals are just multiples of the family's weekly total by 4.3 and 52, respectively.
7. Answers will vary, but probably will center on the kitchen, laundry room, and bathrooms.

## STUDENT PAGE 32

Use *completed Activity Sheet 3* for the following problems.

1. Rank the activities according to the amount of water in gallons for each use.
2. As a class or in a group, choose five of the activities listed on *Activity Sheet 3* and complete a table like the one below, also on *Activity Sheet 4*, for your choices. Using the information on the table, write several sentences comparing water use for these five activities for a total of five days.

**Data Summary of Water Usage**

| Activity | Number of Gallons You Used in 5 Days | Average Number of Gallons Class or Group Used in 5 Days | Difference: *Amount You Used* Minus the *Average Amount Used* |
|---|---|---|---|
| ______ | ______ | ______ | ______ |
| ______ | ______ | ______ | ______ |
| ______ | ______ | ______ | ______ |
| ______ | ______ | ______ | ______ |
| ______ | ______ | ______ | ______ |

3. Make a graph showing your individual water usage for each day of the investigation.
4. Calculate how much water you would use in a week, a month, and a year.
5. Sometimes water is measured in cubic feet. One gallon contains 231 cubic inches.
   a. About how many gallons are in a cubic foot of water?
   b. How many classrooms could be filled up with the amount of water used by your class in a year?
6. In a midwestern city in the United States, the average price for water in 1997 was $2.54 per 1,000 gallons of water. Use this information to calculate about how much your family would spend for water use each week, month, and year.
7. Categorize water usage by rooms in your house. Where is water used the most?

**8.** Answers will vary.

## Practice and Applications

**9–10.** Answers will vary.

## STUDENT PAGE 33

One everyday liquid whose cost you may be familiar with is your favorite cola or other soft drink. Review soft-drink costs in Lesson 3.

Fill in the "Price per Gallon" column in your table for that lesson with an estimate of the price of a gallon of cola in your area.

**8.** Imagine that water cost as much as cola. How much would you spend in a month and in a year?

**Practice and Applications**

**9.** Find a person in your class whose water usage was different from yours. Compare your tables and write a paragraph explaining

  **a.** why your classmate's totals are different from yours.

  **b.** what changes you or your classmate could make to conserve water.

**10.** Write a report ranking and comparing water use for students in your class.

UNIT III

# Using Percents

## LESSON 7

# Percents and Relative Frequency

**Materials:** *Activity Sheet 5,* 1-pound bag of M&Ms, data from Lesson 6
**Technology:** calculator (optional)
**Pacing:** 1 class period

### Overview

This lesson introduces the difference between frequency and relative frequency. Data collection and manipulation continue to be a focus in this lesson.

### Teaching Notes

Several terms introduced in this section are *frequency,* or the number of times an individual event occurs, and *relative frequency,* a proportional measure consisting of the ratio of frequency to the total number of occurrences. Students are asked to think about when to use percents for relative frequencies and when to use fractions. For example, if you were comparing the number of people who voted in the last election in each state, you would use a percent because the state populations are different.

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LESSON 7

# Percents and Relative Frequency

**In a bag of 50 M&Ms, 4 are green and 10 are blue. What percent are green? What percent are blue?**

*Frequency* indicates the number of times something occurs. *Relative frequency* provides a number useful for comparisons because it is a proportional measure obtained by dividing the frequency by the total number of possible occurrences.

$$\text{relative frequency} = \frac{\text{frequency}}{\text{number of possible occurrences}}$$

**OBJECTIVE**

Use relative frequency and data-collection techniques to make predictions.

**INVESTIGATE**

Suppose a student has 50 M&Ms, including 4 green ones. Another student has 60 M&Ms, including 5 green ones. How can you compare the number of green M&Ms each student has? The first student has a relative frequency of $\frac{4}{50}$, or 8%, while the second student has a relative frequency of $\frac{5}{60}$, or about 8.3%. Relative frequency may be written as a fraction, a decimal, or a percent. What are some advantages of expressing relative frequency as a percent? When might it be better to leave it as a fraction?

## Solution Key

### Discussion and Practice

1. Possible answer: If the total numbers of M&Ms in the two packages are different, it would be possible to have the same frequency for one color, but different relative frequencies. For example, 4 M&Ms in a bag of 50 is 8%, and 4 M&Ms in a bag of 55 is 7.3%

2. Possible answer: 6 M&Ms out of 50 is 12%, while 7 M&Ms out of 60 is 11.7%. 7 is greater than 6, but 11.7% is less than 12%.

### Practice and Applications

3. **a.** Answers will vary.

   **b.** Answers will vary.

   **c.** Answers will vary. Increasing the number of packages counted, increasing the size of the bag of M&Ms counted, or a combination of both improves the chances of a good estimate of relative frequency.

STUDENT PAGE 38

**Discussion and Practice**

1. Is it possible for two packages of M&Ms to have the same frequency for one color but different relative frequencies for this color? Explain.

2. Describe a situation in which the number of yellow M&Ms in one bag is greater than a second bag, yet the second bag has greater relative frequency of yellow M&Ms.

**Practice and Applications**

Use the investigation procedure outlined on page 25 and the information you collected in Lesson 6 to answer this question:

Does it appear that the M&M/Mars Candy Company, producers of M&Ms, has a formula in percents for the color mix in M&M packages? What might the formula be for the color mix?

3. You will need the data collected in Lesson 6 on *Activity Sheet* 2 to complete this problem.

   **a.** Copy and complete the table shown below, or use the table on *Activity Sheet 5*. Use your data from Lesson 6 for the first three columns.

**M&M Relative Frequency Color Data**

| | Individual Package Data | | | Class Data | |
|---|---|---|---|---|---|
| Color | Estimated Frequency | Actual Frequency | Relative Frequency | Actual Frequency | Relative Frequency |
| ____ | ____ | ____ | ____ | ____ | ____ |
| ____ | ____ | ____ | ____ | ____ | ____ |
| ____ | ____ | ____ | ____ | ____ | ____ |
| ____ | ____ | ____ | ____ | ____ | ____ |
| ____ | ____ | ____ | ____ | ____ | ____ |
| ____ | ____ | ____ | ____ | ____ | ____ |
| Totals | ____ | ____ | ____ | ____ | ____ |

   **b.** Based on your calculation of relative frequency for the class data, write estimates of the percent of each color in a typical mix of M&Ms.

   **c.** How much variability might there be in your estimates for part b? How could you make your estimates better?

4. Possible answer: Yes;
$\frac{3 \text{ M\&Ms}}{30 \text{ M\&Ms}} = 0.1 = 10\%$ and
$\frac{4 \text{ M\&Ms}}{40 \text{ M\&Ms}} = 0.1 = 10\%$.

5. a. Answers will be the product of each relative frequency and 100.
b. Grids will vary. Emphasize that *percent* means *per 100.* A 10-by-10 grid is a good visual for helping students understand percent.

6. Answers will vary, but should be based on a rough estimate of 533 M&Ms in a package (16 ÷ 0.03).

## Extension

7. a. Answers will vary. Students should discuss whether the relative frequency for each color should be the same in a 1-pound package and a small package of M&Ms.
b–c. You might want to ask the class if they think the same results would have occurred if they had combined enough small packages, about 11, to make a pound.

# STUDENT PAGE 39

4. Is it possible for two packages of M&Ms to have the same relative frequency for one color but a different frequency for this color? Explain.

5. Suppose a package of M&Ms contains 100 candies.
   a. How many candies of each color would you expect to find?
   b. The 10-by-10 grid on *Activity Sheet 5* represents 100 candies in an M&Ms package. Color the grid to represent your predictions for the number of candies of each color.

6. One M&M weighs about 0.03 ounce. About how many green M&Ms would you expect to find in a 1-pound package?

**Extension**

7. Obtain a 1-pound package of M&Ms.
   a. Predict the frequency for each color.
   b. Divide the M&Ms in the package among the students in your class to sort and count the colors. Compile the data.
   c. Write a paragraph explaining how these data compare to the data for small packages.

**Summary**

***Frequency*** is the number of times something occurs. ***Relative frequency*** is a proportional measure obtained by dividing the frequency by the total number of possible occurrences.

$$\text{relative frequency} = \frac{\text{frequency}}{\text{number of possible occurrences}}$$

Relative frequency may be written as a fraction, a decimal, or a percent.

## LESSON 8

# Percents and Visual Comparisons

**Materials:** *Activity Sheets 6* and *7*, grid paper, colored pencils or markers
**Technology:** graphing calculator with statistical capabilities or spreadsheet program (optional)
**Pacing:** 1 class period

### Overview

The purpose of this lesson is to allow students to create different visual representations of percents and to use those representations to make comparisons. Real applications are once again a focus; area and population data for the seven continents are used.

### Teaching Notes

Discuss with students the problems created when a population uses up its resources. Other serious problems include the creation of environmental hazards and destruction of habitats for plants and animals. Students can study the data in the table *Area and Population Data for the Earth* and draw conclusions. However, when the numbers are changed to percents, it is possible to make comparisons between land area and population. This is an important and effective use of percent.

## Solution Key

### Discussion and Practice

1. Answers will vary. Asia, which includes China, is the continent with the greatest population.
2. The population of a country need to be supported by land, farms, industries, and so on, so that people can work and earn enough to provide food and shelter. When one area of the world has a disproportionate part of the population, the land may not be able to support all of the people. The problems this causes are shared by the rest of the world.

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LESSON 8

# Percents and Visual Comparisons

**How many people live in the United States?**

**How many people live in the world?**

OBJECTIVES

Use percents to make comparisons.

Use grids to clarify comparisons.

In the history of the world it took until 1830 for the world population to reach 1 billion people. The population doubled to 2 billion by 1930. It took only 30 years, until 1960, to reach 3 billion. By 1975, the 4 billion mark was reached. Now the total is over 5 billion and still increasing.

INVESTIGATE

When a population uses up its resources, severe problems develop. Several such problems are starvation, illness, and depletion of forests. Can you think of other serious problems?

**Discussion and Practice**

1. Which areas of the world do you think are the most heavily populated?
2. Why do you think it may be important to study population trends?

This lesson will help you look at how populations are distributed on the continents. The population (in thousands) and the area (in thousands of square miles) for each continent are listed in the table on page 41.

3. a. The units for area are 1,000 square miles.
   b. 9,400,000 square miles

4. a–d. See table below.

5. Problems like this provide an excellent opportunity to discuss the problems associated with rounding. To the nearest percent, the area percents are 16%, 12%, 7%, 30%, 20%, 6%, and 9%, for a sum of 100%. However, to the nearest percent, the population percents are 5%, 8%, 9%, 62%, 15%, 0%, 0%, for a sum of 99%. The students in the class may have some suggestions on how to adjust these in an equitable way.

6. Students' grids could be colored so that each percent is represented by the appropriate number of squares. Or students might construct a bar graph of the data.

7. Answers will vary, but paragraphs should show that students understand the disproportionality of the situation.

## STUDENT PAGE 41

**Area and Population Data for the Earth**

| Continent | Area (1,000 sq. mi) | 1990 Population (1,000s) |
|---|---|---|
| North America | 9,400 | 277,000 |
| South America, Latin America, Caribbean | 6,900 | 450,000 |
| Europe | 3,800 | 499,000 |
| Asia | 17,400 | 3,286,000 |
| Africa | 11,700 | 795,000 |
| Australia | 3,300 | 26,000 |
| Antarctica | 5,400 | Uninhabited |
| Entire World | 57,900 | 5,333,000 |

Source: 1990 *World Almanac*

3. Refer to the table above.
   a. What are the units for area?
   b. Write a number that represents the area of North America in square miles.
4. Use the table on *Activity Sheet 6.* The activity sheet has the table above but includes two additional columns.
   a. What percent of the area of the world's continents is represented by the area of North America?
   b. Complete the "Area" column in the table for the rest of the continents.
   c. What percent of the population of the world is represented by the population of North America?
   d. Complete the "Population" column in the table for the rest of the continents.
5. When you add the numbers in the "Percent" column, is the sum exactly 100%? If not, why not?
6. Use *Activity Sheet 6* for this problem.
   a. On the grid labeled "Area," color in the percent of the area of the world's surface occupied by each continent. Use a different color for each continent. Label each area or provide a key.
   b. On the grid labeled "Population," color in the percent of the world's population represented by each continent. Use the same color choices as in part a. Label each area or provide a key.
7. Write a paragraph that compares the amount of land each continent occupies with the size of the population.

**Area and Population Data for the Earth**

| Continent | Area | | 1990 Population | |
|---|---|---|---|---|
| | (1,000 sq. mi) | Percent | (1,000s) | Percent |
| North America | 9,400 | 16.23% | 277,000 | 5.19% |
| South America, Latin America, Caribbean | 6,900 | 11.92% | 450,000 | 8.44% |
| Europe | 3,800 | 6.56% | 499,000 | 9.36% |
| Asia | 17,400 | 30.05% | 3,286,000 | 61.62% |
| Africa | 11,700 | 20.21% | 795,000 | 14.91% |
| Australia | 3,300 | 5.70% | 26,000 | 0.49% |
| Antarctica | 5,400 | 9.33% | Uninhabited | 0.00% |
| Entire World | 57,900 | 100.00% | 5,333,000 | 100.00% |

## Practice and Applications

**8.** **a.** Descending order by area
Asia: 17,400
Africa: 11,700
North America: 9,400
South America, . . .: 6,900
Antarctica: 5,400
Europe: 3,800
Australia: 3,300

**b.** Descending order by population
Asia: 3,286,000
Africa: 795,000
Europe: 99,000
South America, . . .: 450,000
North America: 277,000
Australia: 26,000
Antarctica: uninhabited

**9.** Graphs should correspond to data in the following table.

STUDENT PAGE 42

**Practice and Applications**

**8.** Refer to the table on *Activity Sheet 6*.

**a.** List the continents in descending order by area.

**b.** List the continents in descending order by population.

**9.** Use the first two grids on *Activity Sheet 7*, one to show areas and one to show populations of Canada and the United States. Use the data given below.

**Area and Population of Canada and the U.S.**

| Country | Area (1,000 sq. mi) | 1990 Population (1,000s) |
|---|---|---|
| Canada | 3,849.7 | 28,114 |
| United States | 3,618.8 | 248,710 |

**Extension**

**10.** Use the third grid on *Activity Sheet 7*.

**a.** Use the percent data for area and for population of the continents to make a scatter plot with the percent of Earth's land area on the horizontal axis and the percent of Earth's population on the vertical axis.

**b.** Draw the line $y = x$ and use it to make comparisons.

**c.** Write a short paragraph describing what the plot shows about the relationship between area and population.

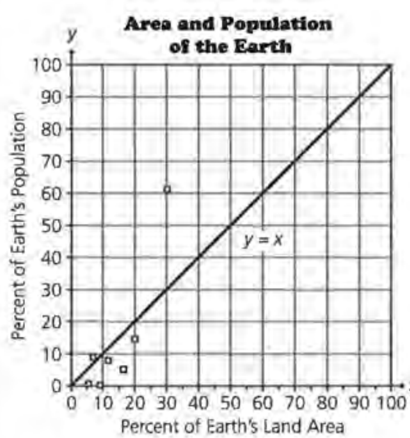
**Summary**

Percent means *per hundred*. To find percent, divide the part by the total and multiply by 100. Percents can be used to compare data that are different, such as area and population.

| Country | Area (1,000 sq. mi) | Percent | 1990 Population (1,000s) | Percent |
|---|---|---|---|---|
| Canada | 3849.7 | 51.5% | 28,114 | 10.2% |
| U.S. | 3618.8 | 48.5% | 248,710 | 89.8% |
| North America | 7468.5 | 100.0% | 276,824 | 100.0% |

## Extension

**10.** Students should see that the $y = x$ line represents the idea of a proportional distribution of area and population. The less a point's vertical distance is to the line, the closer its area and population percents match. The closest continents are Europe, with about 7% of the area of the world and about 9% of its population; and South America, Latin America, and the Caribbean, with about 12% of the land area and 8% of the population.



LESSON 9

# Percents and Surveys

**Materials:** none
**Technology:** calculator
**Pacing:** 1 class period

## Overview

Survey results are often reported using percents. Polls are conducted to answer questions about populations by collecting data from a random sample of a larger population. Percents are used to compare samples and populations of different sizes. This lesson allows students to interpret the results of a survey.

## Teaching Notes

Students are used to hearing about the results of surveys, but they may not have spent much time thinking about how the data are collected. This survey, a Gallup Poll, was conducted by phoning 1,021 adults. After they have read the article, ask students to describe the dialogue they think took place over the telephone in the survey. Then discuss with them the factors that should be considered when deciding on paper or plastic bags and how they would respond to such a survey.

## Follow-Up

Encourage students to share survey reports with the class. Discuss survey methods and interpret the results. Discuss why surveys are important.

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LESSON 9

# Percents and Surveys

**How would you answer the question "Would you prefer a hamburger you ordered to be packaged in a paper wrapper, a foil wrapper, or a box?"**

As environmental problems become more challenging, questions about how to package products are often discussed. One question consumers may consider is whether they prefer to have their groceries packed in paper or plastic bags. Polls are often conducted to help manufacturers and salespeople learn what customers prefer.

**OBJECTIVE**

Interpret the results of a poll using percents.

Read the following article.

**Paper or plastic: Who picks each**

Cox News Service
Washington, D.C.—

A Gallup Poll has found that 48% of American consumers answer "paper" and 37% say plastic when a grocery store checkout clerk asks what kind of bag they want.

The polling organization phoned 1,021 adults to find the answer to this question. It said the percentages were reliable to within plus or minus 3 percentage points.

It found that 83% of the respondents said their stores offered the choice of bag types. About 10% said their grocers offered paper bags only. Only plastic bags were available for about 7% of the respondents.

About one in eight grocery shoppers—13%—use both paper and plastic bags, the poll found. Fewer than 2% gave an answer of anything other than plastic or both.

The study also found that 53% of men use paper bags as compared with 43% of women.

Shoppers under the age of 55 and those with incomes of $25,000 or more also were more likely to prefer paper.

The survey was commissioned by Kraft & Packaging Papers Division of the American Paper Institute, which represents companies that make brown paper bags. In the poll, most of the paper bag users said they regarded the containers as better for the environment. The survey found that 90% reuse the paper bags around their homes.

## STUDENT PAGE 44

### Solution Key

### Discussion and Practice

1. Answers will vary. Use these data to compare the results of a class survey to the results of the Gallup Poll.

2. **a.** The table below records answers to the nearest whole person based on multiplying each percent by 1021.

   **b.** Answers will vary.

But there are environmental problems with both types of bags, Jan Beyea, an official of the Audubon Society, said in an article in the Audubon Activist.

He wrote: "Heretical as it may sound, some uses of virgin paper can be more damaging to wildlife than plastic substitutes. Papermaking pollutes the water, releases dioxin, contributes to acid rain and costs trees."

Plastics are no environmental bargain, Beyea also said. Plastics contaminate the oceans, he said, "they degrade very slowly, they are non-renewable and their production results in pollution."

He urged greater reliance on reusable carriers.

Source: *Milwaukee Journal*, May 3, 1992

**INVESTIGATE**

After reading the article above, what factors do you think should be considered when you decide whether to use paper or plastic bags?

How would you respond if a checkout clerk at a grocery store were to ask whether you prefer paper or plastic bags? Why?

**Discussion and Practice**

1. Poll the students in your class. Record one vote for each student in a table like the one shown below.

**Class Preferences for Paper or Plastic**

| | Paper | Plastic | Both | Other | Total |
|---|---|---|---|---|---|
| Females | ___ | ___ | ___ | ___ | ___ |
| Males | ___ | ___ | ___ | ___ | ___ |
| Totals | ___ | ___ | ___ | ___ | ___ |

2. Set up a table like the one that follows.

**Paper and Plastic Preference Percents and Counts**

| | Paper | Plastic | Other | Total |
|---|---|---|---|---|
| Percent | 48% | 37% | ___ | 100% |
| Number Making This Choice | ___ | ___ | ___ | ___ |

   **a.** In your table, record the results described in the Gallup Poll article.

   **b.** Examine the counts in the two tables you completed. Are the results similar? Why or why not?

**Paper and Plastic Preference Percents and Counts**

| | Paper | Plastic | Other | Total |
|---|---|---|---|---|
| Percent | 48% | 37% | 15% | 100% |
| Number Making This Choice | 490 | 378 | 153 | 1021 |

(2) c. Answers will vary.

3. Possible answers: Which do you prefer for grocery bags, paper or plastic? How many bags of groceries do you usually buy in a typical week? Would you shop at a store that did not have plastic bags available? How would you rate your concern about the impact of plastic bags on the environment, using a scale from 1 (very concerned) to 5 (not at all concerned)?

4. Surveys could be conducted by mail or in grocery stores themselves. The greatest problem with mail-in surveys is that the response rate may be quite small. To make an in-store survey work well, great care would have to be taken to choose stores, times, participants, and so on. An advantage of an in-store survey is that one can observe what people actually choose when they have the option.

## Practice and Applications

5. a. The range of ±3% is a description of the possible error in the survey. The percent who prefer paper, calculated at 48%, may actually vary from 45% to 51%.

   b. Shoppers under the age of 55 and those with incomes of $25,000 or more

6. a. 5,500 paper bags, 7,000 plastic bags, total of 12,500 bags

   b. 44% of bags are paper.

   c. 56% of bags are plastic.

# STUDENT PAGE 45

c. What percent of the males in the class prefer paper bags? What percent of the females in the class prefer paper bags? How do these numbers compare with the information in the article?

3. Write at least three questions you think were asked in the survey discussed in the article.

4. The Gallup Poll discussed in the article was conducted by telephone. Describe other ways data on using paper or plastic could be collected.

**Practice and Applications**

5. Refer to the Gallup Poll article.

   a. The article states that the results were reliable to within plus or minus 3 percentage points. Explain what this means.

   b. According to the article, which groups of people are more likely to use paper bags?

6. At a neighborhood grocery store in Phoenix, Arizona, about 11 bales of paper bags and 7 boxes of plastic bags are used each week. There are 500 paper bags to a bale and 1,000 plastic bags to a box.

   a. What is the total number of bags used each week?

   b. What percent of the bags used are paper?

   c. What percent of the bags used are plastic?

**Extension**

7. *The Universal Almanac*, 1994, reports that in 1991 there were 123,421,000 males and 129,257,000 females in the United States.

   a. Convert these numbers to percent of the total population of the U.S. in 1991. What percent of the total population in 1991 was male? Female?

   b. Assuming that these percents were the same for the 1021 surveyed in the article, about how many of the people surveyed were male? Female?

## Extension

7. a. 252,678,000 was the total population in 1991; 48.8% were male; 51.2% were female.

   b. 498 males, 523 females (499 and 522 if percents are not rounded)

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**Summary**

Polls are conducted to answer questions about populations by collecting data from a random sample of a larger population. Results within a stated range of percentage points from a poll may be a reliable prediction of what could be obtained by surveying the whole population. In most problems, surveying the whole population is impractical, so polls are widely used to obtain approximate results. Percents are used to compare samples and populations of different sizes.

## LESSON 10

# Percents and Diagrams

**Materials:** centimeter ruler, *Activity Sheets 8–10*
**Technology:** graphing calculator
**Pacing:** 1 class period

### Overview

Diagrams may be used to organize data. Once again, percents are used to make comparisons. The data used in this lesson were collected at Rufus King High School in Milwaukee, Wisconsin.

### Teaching Notes

Shot charts provide an organized way to collect data on basketball players. It is important for students to realize that each half of the diagram represents shots attempted/made during a half. An open circle represents an attempt and a miss, and a circle with an × indicates a shot was made. A basket earns 3 points if it comes from outside the large curve and 2 points if it comes from within.

### Follow-Up

Students may collect data on basketball players at their school. The basketball coach or statistician for the team could be invited to discuss the data collected on each player.

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LESSON 10

# Percents and Diagrams

**What kinds of data are used to rate athletic teams or players?**

Data recorded for basketball players include field-goal-completion percent; free-throw-completion percent; time played; scoring average; and number of assists, rebounds, steals, and turnovers.

OBJECTIVES

Use diagrams to find shooting percents of basketball players.

Compare percents.

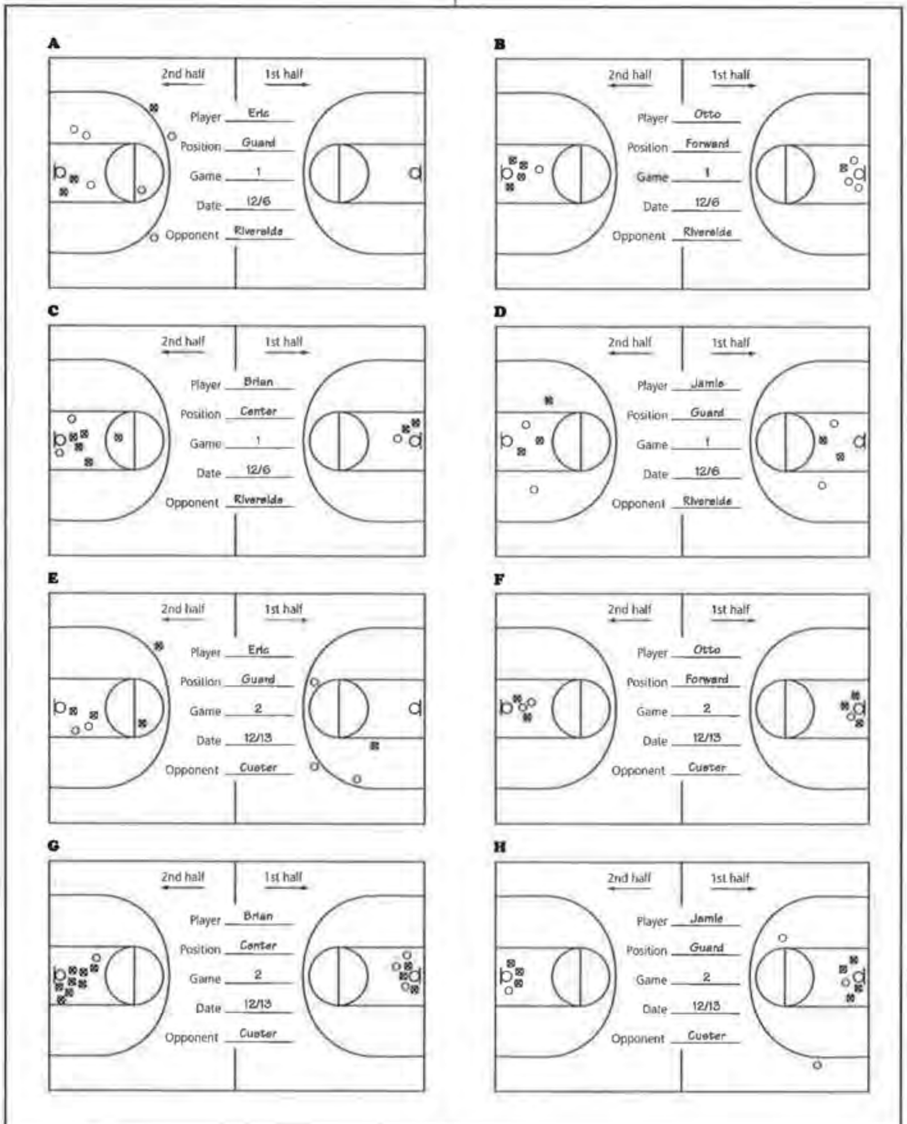
INVESTIGATE

In the 1979–1980 season, the NBA instituted the 3-point-shot rule, and the NCAA followed suit in the 1986–1987 season. High schools also have a 3-point curve. Why are shots made from outside the large curve awarded 3 points instead of 2?

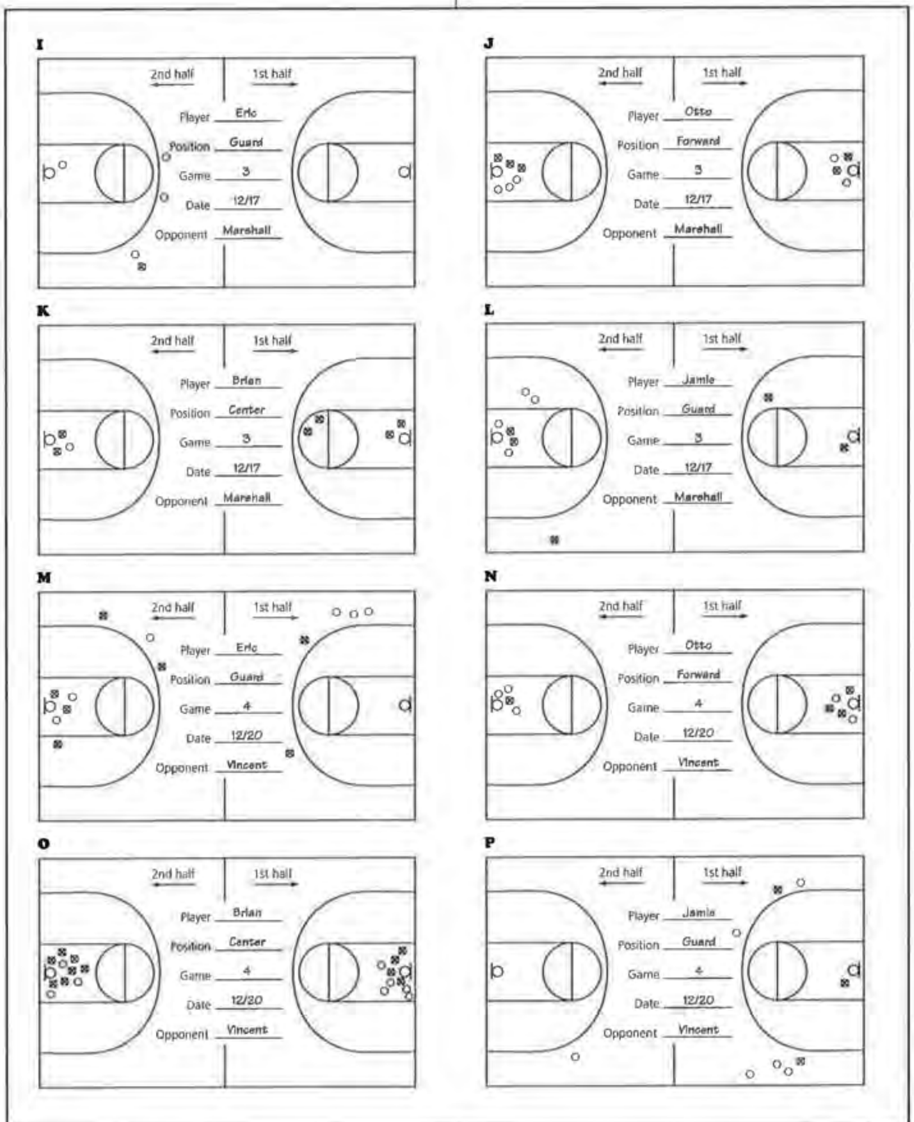
Sometimes data may be displayed in a diagram. Shown on the next two pages are the shot charts of four high-school basketball players: guards Eric and Jamie, forward Otto, and center Brian. These diagrams represent the players' performances in four different games. Each of these players attempted baskets, made a few, and missed a few. They significantly contributed to the final outcome of each game. To interpret the data represented by the charts, you need to keep these general concepts in mind.

- Shots are recorded for the first half of a game at one end of the chart and for the second half at the other end of the chart.
- An open circle on the shot chart indicates that the attempted shot was a miss; a circle with an × through it indicates the shot was made.
- A basket is worth 3 points if the ball is shot from outside the large curve and 2 points if the ball is shot from within the large curve.

## STUDENT PAGE 48



## STUDENT PAGE 49



## Solution Key

## Discussion and Practice

**1.** **a.** 6 shots, 2 shots
**b.** 11 points

**2.** **a.** None
**b.** 10 points

**3.** **a.** 19 shots
**b.** 7 shots

**4.** See page 63.

**5.** **a.** Eric, game 4
Otto, game 2
Brian, game 3
Jamie, game 2

**b.** Answers will vary. A line plot or stem-and-leaf plot would be appropriate. A stem-and-leaf plot is shown.

| | |
|---|---|
| 8 | 6 |
| 7 | 0 3 |
| 6 | 3 4 |
| 5 | 0 0 4 6 6 6 8 |
| 4 | 4 |
| 3 | 3 3 |
| 2 | 0 |

This plot shows the range of percents and that almost half were clustered from 50% to 58%.

# STUDENT PAGE 50

**Discussion and Practice**

1. Look at Shooting Charts A and E.
   a. How many 2-point shots did Eric attempt during the second half of game 1? How many 2-point shots did he make in the second half?
   b. How many points did Eric score in game 2?
2. Look at Shooting Charts F and J.
   a. How many 3-point shots did Otto attempt in game 2?
   b. How many points did Otto score in game 3?
3. Look at Shooting Charts O and P.
   a. How many shots did Brian attempt in game 4?
   b. How many 3-point shots did Jamie attempt in the first half of game 4?

In order to evaluate the performance of each player, it is helpful to organize the data.

4. For each game, complete a table like the following to organize and summarize the data given in the shooting charts. You may use the tables provided on *Activity Sheet 8*.

| Player | Position | 2-Point Shots Attempted | 2-Point Shots Completed | 3-Point Shots Attempted | 3-Point Shots Completed | Percent Completed |
|---|---|---|---|---|---|---|
| Eric | Guard | ____ | ____ | ____ | ____ | ____ |
| Otto | Forward | ____ | ____ | ____ | ____ | ____ |
| Brian | Center | ____ | ____ | ____ | ____ | ____ |
| Jamie | Guard | ____ | ____ | ____ | ____ | ____ |

5. Use your tables from the four games for the following problems.
   a. In terms of percent of shots that were made, identify the best game for each player.
   b. Create an appropriate plot of the percents of shots made for all four players using the data for all four games.

**Game 1**

| Player | Position | 2-Point Shots Attempted | 2-Point Shots Completed | 3-Point Shots Attempted | 3-Point Shots Completed | Percent Completed |
|---|---|---|---|---|---|---|
| Eric | Guard | 6 | 2 | 3 | 1 | 33% |
| Otto | Forward | 9 | 5 | 0 | 0 | 56% |
| Brian | Center | 10 | 7 | 0 | 0 | 70% |
| Jamie | Guard | 9 | 5 | 0 | 0 | 56% |

**Game 2**

| Player | Position | 2-Point Shots Attempted | 2-Point Shots Completed | 3-Point Shots Attempted | 3-Point Shots Completed | Percent Completed |
|---|---|---|---|---|---|---|
| Eric | Guard | 8 | 4 | 2 | 1 | 50% |
| Otto | Forward | 8 | 5 | 0 | 0 | 63% |
| Brian | Center | 15 | 11 | 0 | 0 | 73% |
| Jamie | Guard | 10 | 7 | 1 | 0 | 64% |

**Game 3**

| Player | Position | 2-Point Shots Attempted | 2-Point Shots Completed | 3-Point Shots Attempted | 3-Point Shots Completed | Percent Completed |
|---|---|---|---|---|---|---|
| Eric | Guard | 1 | 0 | 4 | 1 | 20% |
| Otto | Forward | 10 | 5 | 0 | 0 | 50% |
| Brian | Center | 7 | 6 | 0 | 0 | 86% |
| Jamie | Guard | 8 | 4 | 1 | 1 | 56% |

**Game 4**

| Player | Position | 2-Point Shots Attempted | 2-Point Shots Completed | 3-Point Shots Attempted | 3-Point Shots Completed | Percent Completed |
|---|---|---|---|---|---|---|
| Eric | Guard | 5 | 3 | 8 | 4 | 54% |
| Otto | Forward | 9 | 4 | 0 | 0 | 44% |
| Brian | Center | 19 | 11 | 0 | 0 | 58% |
| Jamie | Guard | 1 | 1 | 8 | 2 | 33% |

(5) **c.** Answers will vary.
**d.** Answers will vary.

## Practice and Applications

**6.** 7 out of 17, or about 41%

**7.** Answers will vary. Students could show completed 2-point shots or a combination of completed 2-point and 3-point shots.

**8.** Answers will vary. The player would have to have completed at least one 3-point shot.

**9.** **a.**

| | |
|---|---|
| Eric | 9.75 points per game |
| Otto | 9.5 points per game |
| Brian | 17.5 points per game |
| Jamie | 10.75 points per game |

**b.** 1. Brian
2. Jamie
3. Eric
4. Otto

**10.** Eric has scored 39 points in 4 games. He would have to score 60 points in 5 games to average 12 points per game. He would have to score 21 points in the next game to average 12 points per game.

**11.** **a.** He would have to score 20 points in game 5 to meet his goal. **b.** This is not likely because his average for 4 games is 9.75 points, and the greatest number of points he has scored in a game so far is 18 points.

## Extension

**12.** Eric attempts more 3-point baskets than the other players. In game 4, when he scores 18 points, 12 of the 18 points are from 3-point baskets. He makes about half of the 3-point baskets he attempts. It appears as though Eric's shooting percents for 2-point and 3-point baskets are about the same, and 3-point baskets are worth more.

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**c.** Which player would you say is the best player? Justify your answer.

**d.** Which player(s) seem to be the most consistent? Justify your answer.

**Practice and Applications**

**6.** What percent of his 3-point shots in all four games did Eric make?

**7.** Draw a shot chart for a player who scored 14 points in a game. Use the diagrams on *Activity Sheet 9*.

**8.** Draw a shot chart for a player who scored 15 points in a game and had a shooting percent in the range of 30%–50%. Use the diagrams on *Activity Sheet 9*.

**9.** Another measure of success for basketball players is the average number of points scored per game.

**a.** Find the average number of points scored per game for each player.

**b.** Rank the players according to average number of points scored per game.

**10.** How many points would Eric need to make in the next game to average 12 points per game?

**11.** Eric wants to raise his point average by 2 points in game 5.

**a.** How many points will he have to make in game 5 to meet his goal?

**b.** How likely is his point average to increase by 2 points after game 5? Explain.

**Extension**

What role does distance play in the evaluation of a basketball player?

**12.** Examine Eric's shooting chart carefully and think about the relationship between his distance from the basket and his shooting percent. Write a short summary of your thoughts.

## STUDENT PAGE 52

**13.** Examine Eric's and Otto's completion percents more closely by completing a table like the following or use *Activity Sheet 10.*

**Completion Percents**

| Player | Position | Percent Completed | | | |
|---|---|---|---|---|---|
| | | Game 1 | Game 2 | Game 3 | Game 4 |
| Eric | Guard | _____ | _____ | _____ | _____ |
| Otto | Forward | _____ | _____ | _____ | _____ |

**14.** Use the tables labeled "Eric's Shots" and "Otto's Shots" on *Activity Sheet 10.*

a. Measure the distances from the basket in centimeters or millimeters for each of Eric's and Otto's shots in each game.

b. Find the average distance from the basket for all 2-point shots in each game.

c. Find the average distance from the basket for all 3-point shots in each game.

d. Record the data in the tables on the activity sheet.

**15.** Use the data from the tables in Problem 14 and represent the data in at least one plot or graph.

**16.** Examine the data for Eric and Otto (Problems 14 and 15) and describe the role you think distance from the basket plays in their shooting performance.

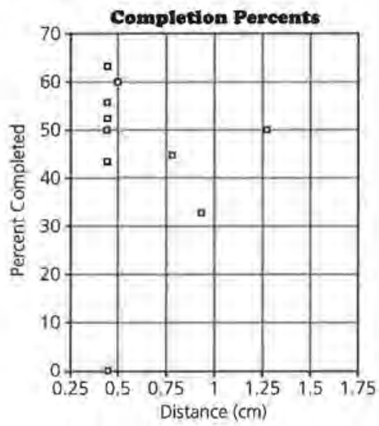
**Summary**

Diagrams and tables may be used to organize data and look for trends, patterns, and relationships. Percents may be used to make comparisons.

**13.** See table below.

**14.** See tables on page 66.

**15.** Answers will vary. A scatter plot of average distances and completion percents for 2-point shots is shown.




**Completion Percents**

| Player | Position | Percent Completed | | | |
|---|---|---|---|---|---|
| | | Game 1 | Game 2 | Game 3 | Game 4 |
| Eric | Guard | 33% | 50% | 20% | 54% |
| Otto | Forward | 56% | 63% | 50% | 44% |

**16.** It appears that, in general, shots made close to the basket have a higher completion percent. A line drawn on the scatter plot slopes down and to the right. Eric's completion percent, however, does not seem to depend on distance. Otto is very successful when shooting a short distance from the basket. It should be noted that Eric and Otto play different positions on the team.

| Eric | Average Distance from Basket | | Percent Completed | |
|---|---|---|---|---|
| | 2-Point Shots | 3-Point Shots | 2-Point Shots | 3-Point Shots |
| **Game 1** | 0.9 cm | 2.5 cm | 33% | 33% |
| **Game 2** | 1.3 cm | 3.1 cm | 50% | 50% |
| **Game 3** | 0.4 cm | 2.6 cm | 0% | 25% |
| **Game 4** | 0.5 cm | 2.4 cm | 60% | 50% |
| **Averages** | 0.8 cm | 2.6 cm | 45% | 41% |

| Otto | Average Distance from Basket | | Percent Completed | |
|---|---|---|---|---|
| | 2-Point Shots | 3-Point Shots | 2-Point Shots | 3-Point Shots |
| **Game 1** | 0.4 cm | 0 cm | 56% | 0% |
| **Game 2** | 0.4 cm | 0 cm | 63% | 0% |
| **Game 3** | 0.4 cm | 0 cm | 50% | 0% |
| **Game 4** | 0.4 cm | 0 cm | 44% | 0% |
| **Averages** | 0.4 cm | 0 cm | 53% | 0% |

## LESSON 11

# Percents and Probability

**Materials:** *Activity Sheet 11,* paper cups
**Technology:** graphing calculator with list capabilities or spreadsheet program (optional)
**Pacing:** $1\frac{1}{2}$ class periods

## Overview

The purpose of this lesson is to give students the opportunity to work with data and determine probabilities. Students once again are analyzing data and making conjectures. With those conjectures, they determine the probabilities of events using percents. Students are able to see that the *frequency* of an event can be used to find the *probability* that it will happen in the future. Percents are useful because comparisons are being made.

## Teaching Notes

The term introduced in this lesson is *probability,* a measure of how likely an event is to occur. Probability may be expressed as a fraction, a decimal, or a percent.

For Problem 10, you may want to emphasize the points made in the previous problems by substituting cups that have been subtly altered. For instance, you could apply a thin layer of white or clear glue on the bottom of each cup. The cups will then be biased with respect to the results the students expected from their previous experimental results.

STUDENT PAGE 53

LESSON 11

# Percents and Probability

**If the probability of rain were 60%, would you carry an umbrella?**

**If the probability of winning a game were 0.15, would you expect to win?**

The *probability* of an event is a measure of how likely it is that the event will occur. Probability is expressed as a value from 0 to 1. A probability of 0 indicates that an event will not occur. A probability of 1 indicates that an event is certain to occur.

OBJECTIVES

Compare the likelihood of outcomes.

State the probability of an event as a ratio or percent.

Determine profit or loss.

INVESTIGATE

The students in Homeroom 207 have decided to sponsor a game booth at the school's fund-raiser. They considered several ideas for games, but they voted to design a "Cup-Toss" game. "We'll have to know what the probabilities are for all of the possible outcomes," Jacqueline said. Jacqueline is the class treasurer. "We want to make money for the school, not give it away."

So the class got some cups and divided up into pairs to collect some data on what happens when a cup is tossed in the air. "I'll bet it lands every time," laughed Chris, the class clown.

"Yeah, and the probability is one that you will goof off every time we have work to do. It's a sure thing," Jacqueline came back.

Why did Jacqueline say that a probability of 1 is a sure thing?

**Discussion and Practice**

The table on page 54 shows the data collected by students in their experiment.

## Solution Key

### Discussion and Practice

1. **a.** Most likely to occur: side; least likely to occur: bottom side down

   **b.** Group 3; group 7

   **c.** One way to compare the results for groups 1 and 3 is to determine the *percents* of the outcomes, since they have a different number of tosses.

   The percents of the different outcomes are fairly close to each other. See table below.

2. Answers will vary, but might include that the cups have to be tossed to a certain height or at least a certain distance and that they must be held in a specified way before the toss. The results should not be determined by the way the cup is held before the toss but should be random.

## STUDENT PAGE 54

**1745 Paper-Cup Tosses in Homeroom 207**

| Groups | Bottom | Top | Side | Totals |
|---|---|---|---|---|
| Group 1 | 7 | 22 | 171 | 200 |
| Group 2 | 11 | 22 | 167 | 200 |
| Group 3 | 12 | 32 | 306 | 350 |
| Group 4 | 11 | 25 | 264 | 300 |
| Group 5 | 10 | 49 | 241 | 300 |
| Group 6 | 12 | 26 | 212 | 250 |
| Group 7 | 2 | 10 | 34 | 46 |
| Group 8 | 1 | 27 | 71 | 99 |
| Totals | 66 | 213 | 1466 | 1745 |

1. Examine the data that Homeroom 207 collected to complete the following problems.

   a. Which outcome seems most likely to occur? Which outcome seems least likely?

   b. Which group had the most cups land on the side? Which group had the least?

   c. Compare the results for groups 1 and 3.

2. Discuss rules for how a cup should be tossed in the Cup-Toss game.

Work with a partner to analyze the cup-toss data from Homeroom 207. Use a table like the one below, provided on *Activity Sheet 11*, or use a spreadsheet.

**Paper-Cup Tosses**

| | Bottom | | Top | | Side | | Totals |
|---|---|---|---|---|---|---|---|
| | Number | Percent | Number | Percent | Number | Percent | |
| Group 1 | ___ | ___ | ___ | ___ | ___ | ___ | ___ |
| Group 2 | ___ | ___ | ___ | ___ | ___ | ___ | ___ |
| Group 3 | ___ | ___ | ___ | ___ | ___ | ___ | ___ |
| Group 4 | ___ | ___ | ___ | ___ | ___ | ___ | ___ |
| Group 5 | ___ | ___ | ___ | ___ | ___ | ___ | ___ |
| Group 6 | ___ | ___ | ___ | ___ | ___ | ___ | ___ |
| Group 7 | ___ | ___ | ___ | ___ | ___ | ___ | ___ |
| Group 8 | ___ | ___ | ___ | ___ | ___ | ___ | ___ |
| Totals | ___ | ___ | ___ | ___ | ___ | ___ | ___ |

| | Bottom | | Top | | Side | |
|---|---|---|---|---|---|---|
| Group 1 | 7 | 3.5% | 22 | 11% | 171 | 85.5% |
| Group 3 | 12 | 3.4% | 32 | 9.1% | 306 | 87.4% |

## STUDENT PAGE 55

3. a. Notice that sums of some groups of percents for top + bottom + side do not equal exactly 100% but are off by 0.1%. You may elect to have your class remedy this by rounding. See table below.

b. The percent table makes it clear that most groups are having comparable results even though they are tossing a different number of times. It also appears that when there are few tosses, the results do not always follow the pattern.

4. a. $\frac{1466}{1745}$, or 84%

b. $\frac{213}{1745}$, or 12.2%

c. $\frac{66}{1745}$, or 3.8%

3. Fill in the "Number" columns in the table. Then, for each group, show the outcomes as percents—comparing the number of times each outcome occurred to the total number of tosses that the group performed.

   a. Find the percent of the total trials that resulted in bottoms, in tops, and in sides.

   b. Does your percent table help you analyze the results better than the tally table? Explain.

4. The probability of an event is a measure of how likely it is to occur. Probability may be expressed as a fraction, a decimal, or a percent.

   a. Based on the experimental data collected in Homeroom 207, what is the probability that a paper cup tossed in the air will land on its side?

   b. What is the probability that a paper cup will land with the top side down?

   c. What is the probability that a paper cup will land with the bottom side down?

**Practice and Applications**

**Profit or Loss?**

Suppose the booth offers a $1 prize for a cup landing on its bottom and a $0.75 prize for landing on its top. There is no prize if a cup lands on its side.

5. Suppose the groups had charged $0.15 per toss.

   a. Would any of the groups have made money?

   b. Would the class have made a profit or lost money on the game? How much?

6. Make up a different fee and prize structure for the cup-toss booth. It should be one that you think would result in more money for the class. Show in a table how much you would expect to take in and pay out using your system.

The cup-toss experiment provides information about the true performance of the cup when it is tossed into the air. It also allows you to make reliable predictions about what will happen in the next 200 or 2,000 tosses. Divide your class into 8 groups to conduct your own experiment like that conducted by Homeroom 207.

**Paper-Cup Tosses**

| | Bottom | | Top | | Side | | Totals |
|---|---|---|---|---|---|---|---|
| | Number | Percent | Number | Percent | Number | Percent | |
| **Group 1** | 7 | 3.5% | 22 | 11.0% | 171 | 85.5% | 200 |
| **Group 2** | 11 | 5.5% | 22 | 11.0% | 167 | 83.5% | 200 |
| **Group 3** | 12 | 3.4% | 32 | 9.1% | 306 | 87.4% | 350 |
| **Group 4** | 11 | 3.75% | 25 | 8.3% | 264 | 88.0% | 300 |
| **Group 5** | 10 | 3.35% | 49 | 16.3% | 241 | 80.3% | 300 |
| **Group 6** | 12 | 4.8% | 26 | 10.4% | 212 | 84.8% | 250 |
| **Group 7** | 2 | 4.3% | 10 | 21.7% | 34 | 73.9% | 46 |
| **Group 8** | 1 | 1.0% | 27 | 27.3% | 71 | 71.7% | 99 |
| **Totals** | 66 | 3.8% | 213 | 12.2% | 1466 | 84.0% | 1745 |

## Practice and Applications

**5.** The data from the table in Problem 3 are used here.

This is an especially good spreadsheet problem since the formula 0.15(number of tosses) – 1(number of bottoms) – 0.75(number of tops) = profit is the same for each group. Clearly groups 5, 7, and 8 lost money.

**a.** The table below shows profit and loss for all groups.

**b.** The total profit made by the class was $36.

**6.** Answers will vary. Compare different groups' plans and discuss reasonableness of different solutions. This is another excellent spreadsheet problem since the formula can be changed very easily allowing your class to play with the numbers. Just reducing the payback for bottoms to $0.75 evens out the experiment dramatically.

| Group | Bottom | Top | Side | Totals | Profit |
|---|---|---|---|---|---|
| 1 | 7 | 22 | 171 | 200 | $6.50 |
| 2 | 11 | 22 | 167 | 200 | $2.50 |
| 3 | 12 | 32 | 306 | 350 | $16.50 |
| 4 | 11 | 25 | 264 | 300 | $15.25 |
| 5 | 10 | 49 | 241 | 300 | –$1.75 |
| 6 | 12 | 26 | 212 | 250 | $6.00 |
| 7 | 2 | 10 | 34 | 46 | –$2.60 |
| 8 | 1 | 27 | 71 | 99 | –$6.40 |
| **Totals** | 66 | 213 | 1466 | 1745 | $36.00 |

7. Answers will vary.

8. See table below for results. The students did improve, as the percents for both bottom and top increased.

9. With these data in mind, it would be a good thing either to lower the paybacks on tops and bottoms, raise the cost of the game, or both.

## Extension

10. If you elect to not alter the cups, have the students do their own manipulating. For instance, they might cut off the upper rim or tape a penny to the bottom (on the inside works best). What might happen to the probability is clear in the case of the penny, but probably will need some experimental testing in the case of the tampered top.

**a–c.** Answers will vary.

# STUDENT PAGE 56

7. Work with a partner or group to collect data.
   a. Toss a paper cup many times. Keep track of your results in table form and compare your data with the data collected by Homeroom 207.
   b. Based on your class data, were the results of Homeroom 207's experiment what you would expect? Support your claim.
8. The students in homeroom 207 did some additional experimenting to see if practice could make them better at the cup-toss game. Their data are shown below. Analyze the data. What results did they find in these experiments? Did they "get better at the game"?

**Room 207 Data**

| Bottom | Top | Side | Totals |
|---|---|---|---|
| 6 | 24 | 70 | 100 |
| 8 | 26 | 66 | 100 |
| 5 | 26 | 69 | 100 |

9. How do these new data change your original estimate of the probabilities for the cup-toss game?

**Extension**

10. Investigate what happens if you tamper with the cup. Figure out a way to change the weight or the size of the cup in such a way that you suspect the probabilities will be affected.
   a. Predict how your change will affect the probabilities of side, top, and bottom.
   b. Do the experiment several times with your group's tampered version of the cup and record your results in table form.
   c. Write a paragraph comparing your group's results with the original results or with the results of other groups.

**Summary**

*Variation* will occur in collecting data. A certain amount of variation is expected. ***Probability*** can be shown as a fraction, a decimal, or a percent. Probability may be used to predict the likelihood of an event. The value of a probability may be any number from 0 through 1.

| | Bottom | | Top | | Side | |
|---|---|---|---|---|---|---|
| | 6 | 6% | 24 | 24% | 70 | 70% |
| | 8 | 8% | 26 | 26% | 66 | 66% |
| | 5 | 5% | 26 | 26% | 69 | 69% |
| **Totals** | 19 | 6.3% | 76 | 25.3% | 205 | 68.3% |

# STUDENT PAGE 57

**Summary for Unit III**

- *Frequency* is the number of times something occurs. ***Relative frequency*** is a proportional measure obtained by dividing the frequency by the total number of possible occurrences.

  $$\text{relative frequency} = \frac{\text{frequency}}{\text{number of possible occurrences}}$$

  Relative frequency may be written as a fraction, a decimal, or a percent.
- Percents may be used to answer questions about data. *Percent* means *per hundred*. To find percent, divide the part by the whole. Percent is an appropriate measure to use to make comparisons.
- Polls are conducted to predict outcomes by collecting data from a random sample of a larger population. Data within a stated range of percentage points from a poll may be reliable as a predictor of the larger population. Percents are used to compare populations of different sizes.
- Diagrams and tables may be used to organize data and look for trends, patterns, and relationships. Percents may be used to make comparisons.
- *Variation* will occur in collecting data. A certain amount of variation is expected.
- ***Probability*** can be shown as a fraction, a decimal, or a percent. Probability may be used to predict the likelihood of an event.
- Data may be collected by making observations, conducting surveys, or performing experiments.

# Assessment for Unit III

**Materials:** none
**Technology:** none
**Pacing:** 1 class period

## Solution Key

1. Possible answer: Determine how well the class performed on a test. Determine how an individual grade (86) compares to the rest of the grades in the class.

2. You may wish to use actual data from class instead of or in addition to the data provided.

   a. The mean score is 84; the median score is 85; the range is 43.

   b.

| Scores | Difference (Score Minus Class Mean) |
|---|---|
| 83 | –1 |
| 78 | –6 |
| 87 | 3 |
| 90 | 6 |
| 93 | 9 |
| 79 | –5 |
| 57 | –27 |
| 85 | 1 |
| 99 | 15 |
| 81 | –3 |
| 85 | 1 |
| 78 | –6 |
| 86 | 2 |
| 64 | –20 |
| 83 | –1 |
| 100 | 16 |
| 86 | 2 |
| 92 | 8 |
| 93 | 9 |
| 86 | 2 |
| 70 | –14 |
| 83 | –1 |
| 83 | –1 |
| 95 | 11 |

   c. No scores are equal to the mean score. The least difference from the mean score is 1. The greatest difference from the mean score is 27. The average difference from the mean is 7 points. A score of 86 is 2 points above the mean and 1 point above the median.

STUDENT PAGE 58

ASSESSMENT

# Assessment for Unit III

Your class has taken a mathematics test. The scores, reported as percents, are listed below.

| | | | |
|---|---|---|---|
| 83 | 78 | 87 | 90 |
| 93 | 79 | 57 | 85 |
| 99 | 81 | 85 | 78 |
| 86 | 64 | 83 | 100 |
| 86 | 92 | 93 | 86 |
| 70 | 83 | 83 | 95 |

Your score, included in the table, was 86%. Your assignment is to explain how well the class did as a whole and how well you did compared to the rest of the class. You should use the investigation process you learned in Unit II.

1. *Problem:* Write a sentence describing the problem.
2. *Data Collection:* The data are listed above. Complete parts a, b, and c so you can consider the center of the data and the variability in the data.
   a. Find the mean, or average, of the data to describe the "center" of the data.
   b. Copy and complete a table like the following for all of the scores.

| Score | Difference (Score Minus Class Mean) |
|---|---|
| 83 | ______ |
| 78 | ______ |

   c. Summarize the information in the difference column to describe the variability in the data set.

## STUDENT PAGE 59

3. Answers will vary. A stem-and-leaf plot is shown.

| | |
|---|---|
| 10 | 0 |
| 9 | 0 2 3 3 5 9 |
| 8 | 1 3 3 3 3 5 5 6 6 6 7 |
| 7 | 0 8 8 9 |
| 6 | 4 |
| 5 | 5 |

4. Answers will vary. Most of the scores clustered in the 80s and low 90s. Two scores were much lower than the others (64 and 57). This can be seen on the stem-and-leaf plot where there is a gap between 70 and the two lowest scores. A score of 86 is slightly above the average but still can be considered in the middle.

5. A, 29%
B, 46%
C, 17%
D, 4%
F, 4%

3. ***Data Presentation:*** Create an appropriate plot or graph of the data.

4. ***Conclusion:*** Write a paragraph that addresses the problem you identified. Justify your conclusion. Include information about the center and the variability of the scores.

5. For the following grading scale, find the percent of As, Bs, Cs, Ds, and Fs.

A 90–100%
B 80–89%
C 70–79%
D 60–69%
F below 60%

UNIT IV

# Measurement Data in Experiments

## LESSON 12

# Exploring Centers and Variability

**Materials:** meter stick, yardstick or tape measure, *Activity Sheets 12* and *13*
**Technology:** graphing calculator (optional)
**Pacing:** 1 class period

### Overview

The purpose of this lesson is to provide students with an opportunity to work with measurement data in a context and to explore the center and the spread of the data. They also have an opportunity to look for bias. Two different activities are provided.

### Teaching Notes

In Unit II, the differences between categorical data and measurement data are first introduced. The colors of M&Ms are an example of categorical data. Measurement data have numerical meaning and can be represented on the real-number line. These data are usually summarized with a measure of center and a measure of spread. It makes sense to add, subtract, and find averages of measurement data. All of the lessons in this unit use measurement data.

The word *center* is used often in daily language. The question "What is the difference between the measure of center of the heights of the players on a basketball team and the height of the center?" is used to help students think about the different ways the word *center* is used. The center or average of the heights is probably not the same as the height of the center, a player on a team. Emphasize that the center of the heights does not have to be the height of any player.

*Bias* may be a difficult concept for students to understand. The approach taken is not to define it but rather to allow students to look for patterns in the data that may indicate bias. An example of bias would be data from a scale that is not set on zero and consistently records a higher weight. Such a scale may be sufficient to measure changes in weight but would not indicate the true weight because the data are biased.

### Follow-Up

Encourage students to continue to look for centers and recognize the mathematical connections.

STUDENT PAGE 63

LESSON 12

# Exploring Centers and Variability

**What is the difference between the measure of center of the heights of the players on a basketball team and the height of the center?**

The measure of center of the heights can be the mean or median of the heights of the players. It is a number that summarizes or averages the heights of the players. It is not necessary for any player on the team to have the center or average height. The height of the center is the height of the player that plays the center position.

Measurement data can take on any value on the real number line. Appropriate units are used to measure an observation, result, or outcome. It is often helpful to summarize measurement data by finding a center.

OBJECTIVE

Explore the concepts of center of measures, variability, and bias.

INVESTIGATE

Have you ever thought about how the weight of a cow is measured? Practically speaking, it is impossible to expect a cow to stand still on a scale while you read the dial. So, while a cow stands on a scale, several measures are recorded over a period of a few seconds and the average or center of the measures is determined to be the weight.

Sometimes measurement data can be biased. ***Bias*** shows favor toward one or more outcomes. The activities that follow show how some factors can affect measurement data. It is important to minimize bias in data collection. It may also be important to notice the variability or spread in a data set.

## Solution Key

### Discussion and Practice

1. Answers will vary. There are several definitions of *center* in the dictionary, many of which are mathematical. The measure of center for a data set can be the mean, median, or mode. The best choice depends on the context.

2. Possible answer: The center of several weight measures is likely to be closer to the true weight.

3. Answers will vary. An example is speed calculated using radar. In Lesson 13, reaction time will be calculated using repeated measures.

### Practice and Applications

4. A doorknob in the classroom that everyone can see would be a good choice. Students should estimate without measuring. The center of the estimates should be calculated for centimeters and inches. Some bias in the data may be apparent. For example, the estimates in inches may be close to a yard because students have an internal concept of that measure. The box plots will allow the students to visualize the variability in the estimates. There may be more variability in the measurement system where students feel least confident. The height of the doorknob should be measured *after* the estimate data are studied.

5. If possible, each group should have a manipulative created from the activity sheets. This activity can be done by passing the manipulative around, allowing students to record their estimate on a piece of paper. The class data should then be examined by creating a box plot of the estimates. Do not expect the center of the estimates to be close to the actual length. A discussion of optical illusions and bias may result.

## STUDENT PAGE 64

**Discussion and Practice**

1. Comment on how the word *center* is used in each of the following:

   Center of a circle
   Center of the universe
   Population center
   Center of mass
   Communication center
   Center of measures

2. Why would it be more reliable to average several weights rather than take the first one recorded when a cow stands on a scale?

3. Describe another situation where the center or average of repeated measures is used.

**Practice and Applications**

4. **Problem A** Estimate the height of the center of a doorknob chosen by the class. Half of the students should estimate the height in inches and the other half should estimate the height in centimeters.
   a. Write your estimate on a piece of paper and give it to your teacher or to a recorder. Record the results on the board or an overhead.
   b. Find a measure of center of the estimates in inches and a measure of center of the estimates in centimeters.
   c. Make a box plot for the inches data and a box plot for the centimeters data.
   d. Describe the variability within each data set.
   e. Measure the height of the center of the doorknob. Determine if one group estimated with greater accuracy than the other. If so, explain why that may have happened.

5. **Problem B** Estimate the length of a given segment using the figure provided on *Activity Sheets 12 and 13*.
   a. Develop and carry out a plan that will result in good data collection in which one student's estimate will not influence others.
   b. Make a box plot of the estimates.

6. Answers will vary. A common way bias can enter a data collection procedure is by not choosing a random sample or a sample that is large enough. In a survey, the wording of questions is very important.

7. a. Mean: 67.5 inches
Median: 67 inches
Mode: 70 inches
b. Jennifer, the tallest player

# STUDENT PAGE 65

c. Describe the variability in the estimates.

d. Measure the correct length of the segment and mark this measure with an X on your box plot.

e. Where does this X fall in relation to your box plot? Describe any factors that may have affected or biased your estimates.

6. Describe some ways that bias can enter into a data-collection procedure for a survey or an experiment.

7. The heights of the players on the Nicolet High School girls' basketball team are listed below.

**Nicolet High School**
**Girls' Basketball Team**

| Number | Name | Height | Year |
|---|---|---|---|
| 12 | Beth | 5'4" | 9 |
| 20 | Liz | 5'6" | 9 |
| 22 | Jessie | 5'6" | 11 |
| 24 | Corey | 5'8" | 12 |
| 32 | Jessica | 5'4" | 12 |
| 34 | Angela | 5'4" | 12 |
| 40 | Kristin | 5'10" | 11 |
| 42 | Caterricka | 5'5" | 9 |
| 44 | Colleen | 5'10" | 9 |
| 52 | Jill | 5'10" | 12 |
| 53 | Shaundra | 5'7" | 12 |
| 54 | Jennie | 5'10" | 12 |
| 55 | Jennifer | 6'2" | 12 |

a. Find the average of the heights of the players on the team.

b. Which girl do you think plays center?

**Summary**

Continuous data are used to find measures such as length, area, volume, and time. Some variation is expected in collecting data. If there is more or less variability in a data set than is likely to have occurred, you may look for sources of *bias*. The mean and median may be used to describe the center of a set of measurement data.

## LESSON 13

# Reaction-Time Experiment

**Materials:** metric ruler, grid paper
**Technology:** graphing calculator (optional)
**Pacing:** 1 class period

### Overview

The purpose of the lesson is to provide an opportunity for students to collect and examine experimental data. Lessons 13 and 14 have the same objectives. You may choose to do either one or both experiments.

### Teaching Notes

Reaction time is an important factor in defensive driving and in some sports. In this experiment, individual reaction time is measured. It is also expected that the class will compare group and male-versus-female data.

It is extremely important to take the time to explain the experimental procedure before students begin their work. Without specific guidelines or rules for holding the ruler, catching the ruler, and measuring the distance, the data could be invalid. The students should agree on what procedures to use and be consistent in following them.

It might help to have students practice dropping a ruler between the fingers of another student and allowing that student to catch the ruler. After students have practiced, have them identify the variables that would contribute to experimental error and decide which techniques should be used.

The investigation process introduced in Lesson 5 will be used for the experiment.

### Follow-Up

Reaction time is important in defensive driving. Factors that may affect reaction time while driving are the use of alcohol, drugs, or a telephone. Ask the students to look for data that show a relationship between the number of accidents and the use of alcohol, drugs, or a telephone while driving.

STUDENT PAGE 66

LESSON 13

# Reaction-Time Experiment

**How quickly do you react when faced with a stimulus?**

**When is a fast reaction time important?**

OBJECTIVES

Collect and interpret experimental data to measure reaction time.

Use measurements in the investigation process.

Your *reaction time* is a measure of the time it takes you to respond to a stimulus. Do males react faster than females do? The experiment in this lesson studies reaction time by dropping a ruler through the fingers of a student and measuring how far the ruler falls before it is caught.

## INVESTIGATE

Before doing this experiment, some decisions need to be made about what methods or techniques will be used to minimize experimental error. Read the experimental procedure below. Then try the procedure a few times and decide which dropping-and-catching techniques should be controlled. Consider these variables:

- How far above the hand should the ruler be held?
- How many fingers should be used to catch the ruler?
- Should the measurement be made above or below the fingers?

Are there other important variables to consider?

### Experimental Procedure

Work in pairs to conduct this experiment.

- One of you holds a metric ruler and drops it through the open fingers of the other.
- Measure how far the ruler falls before it is caught.
- Each of you should complete the procedure ten times.
- Record the data in a table.

## Solution Key

### Discussion and Practice

1. Students should be allowed to choose their own methods for recording distances. Some may create a table, and others a graph. Their observations may be affected by their recording methods. The observations can be very interesting and should be discussed.

2. This question asks for an appropriate plot. Allow the students to make some decisions regarding which plots they choose. A center distance for each student must be calculated.

## STUDENT PAGE 67

**Discussion and Practice**

**Problem:** How long does it take you to react when a ruler is dropped through your fingers? Do males in the class react faster than females do?

*Data Collection*

1. a. Perform the experiment ten times with one person catching the ruler. The person who drops the ruler will also record the distances the ruler fell each time.

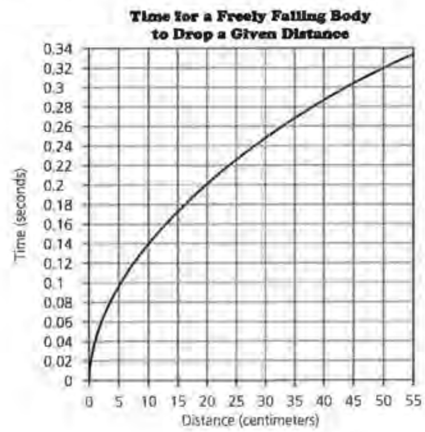
   b. Reverse roles and perform the experiment ten more times.

   c. Write at least three observations about each set of ten drop distances.

*Data Presentation*

2. Plot the data sets separately in an appropriate way and calculate a center distance for each person using mean, median, or mode.

Pictured below is the graph "Time for a Freely Falling Body to Drop a Given Distance." Use it to convert your distance to a reaction time. For example, if the ruler fell through your fingers a distance of 20 cm before you caught it, the time it took you to react and catch the ruler was 0.2 second.



STUDENT PAGE 68

**3.** Convert the center distance to a measure of reaction time. Record it with "F" for female and "M" for male. Record all the results on the board or an overhead projector.

**4.** On one set of axes, make two box plots, one for female and one for male reaction-time data.

*Conclusion*

**5.** Write a paragraph about whether males appear to have faster or slower reaction times than females do.

**Practice and Applications**

**6.** Examine your own data a little more closely by making a different plot of your drop distances than the one you made in Problem 3. For example, if you made a plot over time, you could make a box plot or a stem-and-leaf plot.

**a.** Describe any trends or patterns in your reaction-time data.

**b.** Predict the results of ten or more tosses.

**7.** Describe the variability in your data set.

**8.** If this experiment were done with the catcher blindfolded and the dropper giving a verbal or tactile clue, do you think the results would be different? What would you record if the person missed the ruler entirely?

**9.** How do you think using your opposite hand would affect results? Explain.

**Extension**

**10.** The distance it takes a car to stop depends on the reaction time of the driver and the speed of the car. The reaction time is the interval between when the driver recognizes the car must stop and the brakes are applied. In the time it takes a driver with normal reaction time to react to an emergency, a car travels a distance of $d$ feet, where $d = 1.1$ times the speed of the car. Find the distance in feet a car going the given speed travels before the driver hits the brakes.

**a.** 35 mph **b.** 65 mph

**Summary**

We conduct experiments to collect data to answer a question or solve a problem. In an experiment, variables that may affect results should be considered. Some variables may be controlled.

**3.** Since reaction time is being studied, the graph is used to convert distance to time.

**4.** Answers will vary. These box plots can be made using the graphing calculator and examined together.

**5.** Remind students to base their conclusions on the data.

## Practice and Applications

**6.** Answers will vary. Emphasize that different patterns may emerge by looking at data represented in various ways. Using data to predict future patterns is important in some areas. Examples are weather forecasting or predicting ranks for sports teams.

**7.** Answers will vary. The variability may be interesting. One could argue that students get tired after a certain number of trials or that they begin to perform better. Either argument contributes to the variability.

**8.** Answers will vary.

**9.** Answers will vary. Reaction time will most likely increase if a right-handed person uses his or her left hand.

## Extension

**10. a.** 38.5 feet
**b.** 71.5 feet

## LESSON 14

# Burst-Your-Bubble Experiment

**Materials:** solution to make bubbles, straws, ruler, newspaper or paper towels, grid paper
**Technology:** graphing calculator (optional)
**Pacing:** 1 class period

### Overview

The purpose of the lesson is to provide an opportunity for students to collect and examine experimental data. Lessons 13 and 14 have the same objectives. You may choose to do either one or both experiments.

### Teaching Notes

This class estimate should be determined before the experiment is conducted. The actual data will be compared to the class estimate at the end of the lesson. Again, the investigation process introduced in Lesson 5 will be used for the experiment. Cover the desks and then allow students to practice blowing bubbles.

Two possible recipes for bubble solution follow.

| | |
|---|---|
| 1 gallon water | 1 gallon water |
| $\frac{1}{2}$ cup liquid dishwashing detergent | $\frac{2}{3}$ cup liquid dishwashing detergent |
| $\frac{1}{4}$ cup glycerin | 1 tablespoon glycerin |

## Solution Key

### Discussion and Practice

1. Methods for measuring the diameter of bubbles should be discussed, since accurate measurements may be tricky. One method is to measure the bubble on the straw. Another method is to allow the bubble to burst on the newspaper and then measure the film it leaves. Students may use different methods.

2. After a certain amount of practice time, students should begin measuring the diameters of their bubbles. Answers will vary.

3. Students should choose their own method for recording diameters.

STUDENT PAGE 69

LESSON 14

# Burst-Your-Bubble Experiment

**How large a bubble do you think you can blow with the "bubble stuff" you used as a young child?**

INVESTIGATE

The experiment below will help you answer the question "How large a bubble can you blow?" Estimate the diameter of a bubble you could blow using a straw. Decide as a class whether you would estimate in centimeters or inches. Share your estimates and find a reasonable class estimate for the size of a bubble.

OBJECTIVES

Collect experimental data.

Use measurements in the investigation process.

**Discussion and Practice**

Work with a partner. You will need bubble mixture, a straw, and a ruler. Take your books off your desk. If you don't want to get bubble stuff on your desk, cover it with paper towels or newspaper. Lay the ruler on your desk.

1. Decide how you might measure a bubble's diameter.
2. Put a straw in the bubble mixture and blow a bubble. Find the diameter of the sphere you have blown by reading the measurement from the ruler. You may have to do this quickly, perhaps through the bubble while it is still attached to the end of your straw.
3. Blow at least 5 bubbles and record their diameters.

Complete the investigation process.

**Problem: What is the average diameter of the bubbles you can blow with a straw?**

4. The data-collection table will be used to make the plots for Problem 5.

5. Students should examine different plots of their data before choosing an appropriate one.

6. Answers will vary.

## Practice and Applications

7. Compare the actual data to the class estimate. Discuss factors that may have influenced the estimate or the data-collection process.

8. Possible questions: What is the size of the largest bubble? Smallest bubble? Did the bubbles increase in size over time? What is the volume of the average bubble? What is the surface area of the average bubble?

9. Answers will vary.

## Extension

10. Encourage students to be creative, but the poster should include a statement of the problem, a description of the data-collection procedures, the results (data tables, graphs), and the conclusions.

# STUDENT PAGE 70

4. *Data Collection*

   Collect data from students in your group. Each student should make at least 5 bubbles. Organize your data in a table or chart that can be turned in with your conclusions.

5. *Data Presentation*
   a. Calculate a measure of center using mean or median.
   b. Create an appropriate graph or plot of your data.

6. *Conclusion*
   a. Summarize your results. Include a measure of center and a description of the variability in your data.
   b. Explain any problems that you had or things you could have done to make your investigation easier.

**Practice and Applications**

7. Compare your group's answer to the class estimate.

8. Identify another problem or question that the investigation process could be used to answer.

9. The volume of your bubbles can be calculated using the formula volume = $\frac{4}{3}\pi r^3$, where $r$ = radius, or $\frac{1}{2}$ the diameter. Find the volume of your average bubble.

**Extension**

10. Make a poster describing your experiment.

**Summary**

- The investigation process includes:
  - Identification of the problem
  - Data collection
  - Data presentation
  - A summary of results
- Tables, charts, graphs and plots may be used to organize experimental data.
- The summary may include information about the center of the data and the variability in the data.

# STUDENT PAGE 71

**Summary for Unit IV**

- Continuous data are used to find measures such as length, area, volume, and time. Some variation is expected in collecting data. If there is more or less variability in a data set than is likely to have occurred, you may look for sources of *bias*. ***Bias*** shows favor toward one or more outcomes. The mean and median may be used to describe the center of a set of measurement data.
- Experiments are conducted to collect data to answer a question or solve a problem. Variables that may affect the results should be considered. Some variables may be controlled.
- The investigation process includes:
  - Identification of the problem
  - Data collection
  - Data presentation
  - A summary of results
- Tables, charts, graphs and plots may be used to organize experimental data.
- The summary may include information about the center of the data and the variability in the data.

PROJECT

# Waiting Time in the Lunch Line

**Materials:** grid paper, *Activity Sheets 14* and *15*
**Technology:** graphing calculator (optional)
**Pacing:** 2 class periods

## Overview

The purpose of the lesson is to provide an activity where students can use the investigation process and the skills they have learned about collecting and analyzing data. The investigation process allows students to quantify information to answer a question or address a problem.

## Teaching Notes

Lunch-line procedures often vary in different schools, but the problems are usually the same. A method must be devised so that several hundred students can eat lunch in a short period of time. If students have to spend a large percent of their lunch time standing in line, they may become frustrated. Even if there are no problems, the data may be interesting to study. More efficient methods for serving students or longer lunch periods may be reasonable suggestions after the investigation is completed. Emphasize that a process that is based on data, carefully planned and carried out, provides a very powerful argument.

Students need to be able to collect data during the lunch hours. This may be done outside of class.

Permission from the administration may be needed to conduct the survey.

## Solution Key

### Discussion and Practice

1. Allow students to address the questions and decide how to handle the variables. They could first discuss them in small groups and then come to a consensus as a class. The procedures for conducting the survey should be carefully thought out and written down so that everyone understands them.

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PROJECT

# Waiting Time in the Lunch Line

**What is the longest time you have spent waiting in line?**

**What are some things for which people wait in line?**

Americans spend hours waiting in line. Lines form at the checkout counter, the box office, the bank, the post office, the park, and the classroom door. People even wait in line on the telephone. Lines provide an orderly system for serving people. Sometimes statisticians study the amount of time customers must wait in line for service.

INVESTIGATE

One of the places in your school where you may have to wait in line is the lunchroom. The amount of time you wait in line may affect how long you have to eat or talk to your friends.

This project involves collecting data and analyzing it to find out how much time students spend waiting in the lunch line(s) at your school.

The problem you are to address is: **How much time do students spend waiting in the school lunch line?**

**Discussion and Practice**

1. Identify the variables related to the problem and develop a plan for collecting data.
   a. When should the data be collected and for how many days?
   b. Are the lines longer when certain foods are served?

2. At the end of the investigation, students will be able to compare the actual data with their estimates.

3. Have students use the cards from *Activity Sheet 14* to collect data. These could be handed to students when they begin their waiting time. The start time should then be recorded. Another person could collect the cards at the end of the waiting time and record the end time. It may be reasonable to color-code the cards for different lines, different lunch periods, or different days.

4. A data summary may be completed for each lunch line, each day, or each period so that comparisons can be made. Students should consider measures of center and look at variability. Even if the measure of center is a reasonable wait time, the variability may not be acceptable.

## STUDENT PAGE 73

c. What categories should be included? Are there different lunch lines and different lunch times? Will students who bring their lunches and don't stand in line at all be considered?

d. How should wait time be defined?

e. How should the data be collected? One possibility is to hand each student a card as students get in line. On the card, the specific line and a time indicating the beginning of the wait time is recorded. At the end of the waiting time, this card is handed back to a data collector and the end of waiting time is recorded. You might devise and use cards like the one shown below or cut out the cards on *Activity Sheet 14*.

**Lunch-Line Waiting Time**

Date ____________________

Line ____________________

Start Time ____________________

End Time ____________________

2. Before beginning the collection of data,

   a. estimate the average amount of time you spend waiting in the lunch line on a school day.

   b. estimate the percent of your total lunch time you spend waiting in line.

3. Collect waiting-time data for at least two days. Set up a table or use the one on *Activity Sheet 15* to compile the data.

4. Summarize the data using the following procedure.

   a. Make at least one plot of the data.

   b. Describe any patterns you see in the data.

   c. Calculate the mean waiting time and the median waiting time for each day you collected data. Compare them.

   d. Find the range of waiting times.

5. A review of percents may be needed.

6. Answers will vary.

7. Answers will vary.

8. Answers will vary.

## Extension

9. Answers will vary.

10. Answers will vary.

## STUDENT PAGE 74

5. Analyze the data.
   a. Convert the waiting-time data to percents of total lunch time students spend standing in line.
   b. Make a box plot of the percents.
   c. How do the actual percents compare to your estimate from Problem 2?
6. What is the "typical" daily waiting time in the lunch line for students in your school? Consider measures of center and variability in answering this question.
7. Do you think the amount of time scheduled for lunch is about right, too short, or too long? Use your results to support your answer.
8. Write a letter to the administration of your school that includes the results of your investigation.

**Extension**

9. Present your results to the administration or student government at your school.
10. Identify some other issues in your school that might be resolved by using a similar investigation process. An example is the amount of time scheduled between classes.

# Teacher Resources

## Context and Units

NAME ______________________________

1. List two situations in which 20 is a large number. Specify the units in each case.

2. What data do you think would be important to include in determining an academic rating for your high school? List at least five different items.

Use the following table for Problems 3–5.

**Yearly Consumption of Soft Drinks by Region**

| Region | Population (millions) | Total Gallons (millions) | Per-Capita Consumption (gallons) | Retail Sales (millions of dollars) |
|---|---|---|---|---|
| New England | 12.8 | 554.5 | 43.3 | 2,076.9 |
| Middle Atlantic | 37.6 | 1,737.9 | 46.2 | 6,936.5 |
| East North Central | 42.1 | 2,033.3 | 48.3 | 7,518.0 |
| West North Central | 17.8 | 868.4 | 48.8 | 2,941.8 |
| South Atlantic | 42.8 | 2,152.8 | 50.2 | 7,181.0 |
| East South Central | 15.4 | 845.6 | 54.9 | 2,862.5 |
| West South Central | 27.1 | 1,303.8 | 48.1 | 4,570.5 |
| Mountain | 13.4 | 496.8 | 37.1 | 2,013.2 |
| Pacific | 37.6 | 1,538.2 | 40.9 | 6,268.8 |
| **Totals** | 246.8 | 11,531.2 | 46.7 | 43,106.2 |

Source: *Beverage World*, May, 1990

3. How many gallons of soda are consumed each year in the United States?

4. List the region that has the greatest number in each column.

a. Total Gallons

b. Per-Capita Consumption

c. Retail Sales

5. Explain why the region with the greatest per-capita number can be different from the region with the greatest total-gallons number.

6. The following table provides information for strawberry fruit pops.

| Brand | Cost per Serving (cents) | Serving Size (ounces) | Calories per Serving |
|---|---|---|---|
| Dole Fruit'N Juice | 47 | 2.5 | 70 |
| Dreyer's Edy's Tropical Fruit Bars | 47 | 3 | 90 |
| Dole Fruit Juice | 24 | 1.75 | 45 |
| Dole Fruit Juice No Sugar Added | 24 | 1.75 | 25 |
| Frozfruit Chunky | 69 | 4 | 80 |
| Popsicle Juice Sets | 20 | 1.6 | 45 |
| Welch's Fruit Juice Bars | 25 | 1.75 | 45 |
| Minute Maid All Natural Juice Bars | 24 | 1.75 | 50 |
| Popsicle All Natural | 18 | 1.75 | 45 |
| Welch's Light | 25 | 1.75 | 25 |

Source: *Consumer Reports,* July, 1996

a. What is the average cost per serving?

b. Calculate the cost per ounce for each brand and arrange those numbers in order from least to greatest. Which brand is the best buy?

c. How much would you expect to pay if you wanted to buy enough strawberry pops to serve 12 people? Express your answer as a range.

d. Describe the variability in the number of calories.

# Sorting and Counting

NAME ______________________________

1. List the measures or categories you would use to summarize each data set.
   a. Eye color
   b. Travel time to school in the morning
   c. Measures of the length of a board

2. Consider the numbers listed. Suppose all of the numbers are used in each situation below. Summarize the data in appropriate ways.

   4, 6, 10, 12, 18, 24, 30, 36

   a. The numbers represent the number of years of experience job applicants have.
   b. The numbers represent the available television channels in a local viewing area.

3. The following problems will lead you through the investigation process to answer these two questions.
   - What sums occur most frequently when two dice are tossed?
   - If two dice are tossed 36 times, what is the range for the number of times you would expect sums of 7 and 12?

*Data Collection*

Six trials were conducted. In each trial, two dice were tossed 36 times.

| Sum | Trial 1 | Trial 2 | Trial 3 | Trial 4 | Trial 5 | Trial 6 |
|---|---|---|---|---|---|---|
| 2 | 0 | 1 | 0 | 0 | 1 | 4 |
| 3 | 3 | 4 | 0 | 4 | 5 | 0 |
| 4 | 3 | 2 | 4 | 2 | 2 | 2 |
| 5 | 3 | 6 | 5 | 5 | 3 | 9 |
| 6 | 6 | 4 | 4 | 7 | 4 | 5 |
| 7 | 4 | 5 | 7 | 5 | 9 | 3 |
| 8 | 6 | 7 | 6 | 4 | 1 | 3 |
| 9 | 5 | 4 | 5 | 2 | 5 | 4 |
| 10 | 4 | 2 | 3 | 3 | 1 | 4 |
| 11 | 1 | 1 | 1 | 3 | 1 | 2 |
| 12 | 1 | 0 | 1 | 1 | 4 | 0 |

*Data Presentation*

**a.** Create a visual summary of one or more trials or the entire data set.

**b.** Calculate appropriate numerical summaries of the data such as measures of center or spread.

*Conclusion*

**c.** Write a paragraph describing your conclusions about the data to answer the two questions above.

**4.** The table on page 101 lists the number of times each sum from 2 to 12 would be expected in 36 tosses of the dice. Find a *closeness score* for each trial in Problem 3 to determine which trial was closest to the expected numbers.

**Difference Between Actual Number and Expected Number**

| Sum | Expected Number | Trial 1 | Trial 2 | Trial 3 | Trial 4 | Trial 5 | Trial 6 |
|---|---|---|---|---|---|---|---|
| 2 | 1 | ______ | ______ | ______ | ______ | ______ | ______ |
| 3 | 2 | ______ | ______ | ______ | ______ | ______ | ______ |
| 4 | 3 | ______ | ______ | ______ | ______ | ______ | ______ |
| 5 | 4 | ______ | ______ | ______ | ______ | ______ | ______ |
| 6 | 5 | ______ | ______ | ______ | ______ | ______ | ______ |
| 7 | 6 | ______ | ______ | ______ | ______ | ______ | ______ |
| 8 | 5 | ______ | ______ | ______ | ______ | ______ | ______ |
| 9 | 4 | ______ | ______ | ______ | ______ | ______ | ______ |
| 10 | 3 | ______ | ______ | ______ | ______ | ______ | ______ |
| 11 | 2 | ______ | ______ | ______ | ______ | ______ | ______ |
| 12 | 1 | ______ | ______ | ______ | ______ | ______ | ______ |
| **Closeness Scores** | | ______ | ______ | ______ | ______ | ______ | ______ |

**5.** Study the relationship between the *mean* of each sum in Problem 3 and the number of times each sum from 2 to 12 would be expected in 36 tosses of the dice.

**a.** Find a closeness score to compare the means to the expected sums.

**b.** Does the closeness score for the means indicate that the means are closer to the expected sums than one of the six trials?

**c.** Predict the closeness score for means of 50 trials.

# Using Percents

NAME ______________________________

**1.** The table shows the percent of different materials Americans throw away. Create a visual display of the information using the grid below.

| Waste Products | Percent |
|---|---|
| Paper and Cardboard | 37% |
| Glass | 7% |
| Metals | 8% |
| Plastics | 8% |
| Rubber and Leather | 2% |
| Textiles and Wood | 9% |
| Food Wastes | 7% |
| Yard Wastes | 18% |
| Other | 4% |

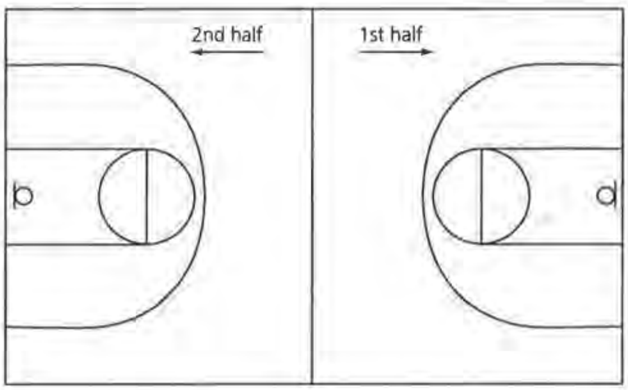
Source: Environmental Protection Agency

2. Suppose 150 million tons of garbage are thrown away each year in the United States.

a. How many tons of plastics are thrown away?

b. How many tons of paper and cardboard are thrown away?

3. Draw a shot chart for a player who scored 11 points in a game and had a shooting percent in the range of 30% to 50%.




4. A thumbtack was tossed by four different groups. Each time, it landed either head down or on its side. Complete the following table.

| Groups | Landing Head Down Number | Landing Head Down Percent | Landing on Side Number | Landing on Side Percent | Totals |
|---|---|---|---|---|---|
| Group 1 | 27 | ____ | 23 | ____ | 50 |
| Group 2 | 10 | ____ | 23 | ____ | 33 |
| Group 3 | 14 | ____ | 14 | ____ | 28 |
| Group 4 | 34 | ____ | 31 | ____ | 65 |
| Totals | ____ | ____ | ____ | ____ | ____ |

5. Based on the experimental data in your table, what is the probability a thumbtack will land

a. head down?

b. on its side?

6. Your class has taken a 15-point quiz. The scores are listed below.

| | | |
|---|---|---|
| 12 | 15 | 10 |
| 9 | 8 | 13 |
| 10 | 11 | 12 |
| 15 | 9 | 7 |
| 5 | 9 | 12 |
| 14 | 15 | 11 |

Your score, included in the table, was 11. Your task is to explain how well the class did as a whole and how well you did compared to the rest of the class.

a. Write a sentence describing the problem.

b. Create an appropriate plot or graph of the data.

c. Write a paragraph with your conclusion. Include information about the center and the variability of the scores.

d. For the following grading scale, find the percent of As, Bs, Cs, Ds, and Fs. Round each percent to the nearest whole number.

| | |
|---|---|
| A | 90–100% |
| B | 80–89% |
| C | 70–79% |
| D | 60–69% |
| F | below 60% |

# Measurement Data in Experiments

NAME ______________________________

These data, represented in a stem-and-leaf plot, were given as estimates for the height of a doorknob in a classroom.

**Centimeters**

| | |
|---|---|
| 3 | 0 6 |
| 4 | |
| 5 | 0 5 6 |
| 6 | 0 0 5 |
| 7 | 5 |
| 8 | 4 4 |
| 9 | 0 0 0 |
| 10 | 0 0 0 0 0 0 0 |
| 11 | 5 |
| 12 | 0 |
| 13 | 0 |
| 14 | |
| 15 | 0 0 |
| 16 | 0 |
| 17 | |
| 18 | |
| 19 | |
| 20 | 0 0 0 |
| 21 | |
| 22 | |
| 23 | |
| 24 | |
| 25 | |
| 26 | |
| 27 | 5 5 |
| 28 | |
| 29 | |
| 30 | 0 |

**Inches**

| | |
|---|---|
| 1 | 5 8 |
| 2 | 4 4 4 6 8 |
| 3 | 0 0 0 1 2 2 4 4 6 6 6 6 6 6 6 6 7 8 8 8 |
| 4 | 0 0 0 2 2 2 2 4 5 6 |
| 5 | 0 0 |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | 0 |

1. Find the measure of center of the estimates
   a. in centimeters.
   b. in inches.

2. Make a box plot for the data
   a. in centimeters.
   b. in inches.

3. Describe the variability within the data set
   a. in centimeters.
   b. in inches.

4. The actual height of the doorknob is 94 centimeters or 37 inches. Determine if one group estimated with greater accuracy than the other. If so, explain what may have happened.

5. A student performed a reaction-time experiment and recorded the following distances in centimeters. The distances were recorded in the order they were measured.

   7.2, 13, 14, 9.1, 8.9, 7.7, 10.1, 10.7, 13, 9.1

   a. Create at least two different plots for the data.
   b. Calculate a *center* distance.
   c. Describe the variability in the data set.

## Mathematics in a World of Data

NAME ______________________________

**1.** Katie estimated life expectancy for the five animals shown in the table. The actual life expectancy for each animal is also given.

| Animal | Estimated Life Expectancy (years) | Actual Life Expectancy (years) |
|---|---|---|
| **Lion** | 15 | 10 |
| **Mouse** | 1 | 2 |
| **Parakeet** | 2 | 8 |
| **Bear** | 10 | 22 |
| **Elephant** | 21 | 35 |

**a.** Make a scatter plot of the data with estimated life expectancies on the horizontal axis and actual life expectancies on the vertical axis.

**b.** Explain how the scatter plot can be used to compare the estimates to the actual life expectancies.

**2.** The table below gives Katie's estimates as well as those made by Mandy, Rachel, Julia, and Jon.

| Animal | Katie's Estimates | Mandy's Estimates | Rachel's Estimates | Julia's Estimates | Jon's Estimates | Actual Life Expectancy |
|---|---|---|---|---|---|---|
| Lion | 15 | 12 | 20 | 20 | 13 | 10 |
| Mouse | 1 | 4 | 3 | 1 | 6 | 2 |
| Parakeet | 2 | 3 | 7 | 5 | 5 | 8 |
| Bear | 10 | 15 | 30 | 25 | 18 | 22 |
| Elephant | 21 | 20 | 40 | 40 | 20 | 35 |
| Closeness Scores | ______ | ______ | ______ | ______ | ______ | ______ |

**a.** Compare Katie's estimates with those of Mandy, Rachel, Julia, and Jon by calculating a closeness score for each person.

**b.** Explain how you calculated your closeness scores.

**c.** Which person had estimates closest to the actual life expectancies?

**3.** The heights of the Nicolet girls' basketball team are recorded in the table.

| Player | Height |
|---|---|
| Ann | 5'5" |
| Lauren | 5'6" |
| Lindsay | 5'7" |
| Kimberly | 5'9" |
| Jessi | 5'4" |
| Caterricka | 5'5" |
| Robin | 5'7" |

| Player | Height |
|---|---|
| Katie | 5'1" |
| Colleen | 5'10" |
| Sarah | 5'11" |
| Beth | 5'6" |
| Elizabeth | 5'4" |
| Katherine | 5'5" |
| Jenay | 5'11" |

**a.** Make a box plot of the data.

**b.** Find a measure of center for the data.

**c.** Describe the variability in the data set.

**4.** The cost of cereal is affected by more than what goes into the box. The table below shows how the cost of a box of Cheerios sold for $3.29 can be divided.

| | Cost (cents) | Percent of Cost |
|---|---|---|
| **Profit** | 56 | ______ |
| **Advertising** | 42 | ______ |
| **Coupons** | 82 | ______ |
| **Labor** | 19 | ______ |
| **Materials** | 82 | ______ |
| **Other** | 48 | ______ |

Source: *Zillions for Kids from Consumer Reports,* February/March, 1996

**a.** Complete the Percent of Cost column in the table. Round the percents to the nearest whole number.

**b.** What percent covered the cost of materials?

**c.** What percent went toward advertising and coupons?

**d.** Create a visual display of the information on the grid below.

5. Use the investigation process to answer these questions.

- **How heavy are the backpacks students carry?**
- **Do students carry backpacks that are too heavy for them?**

*Data Collection*

The following data were collected about the weights of students and their backpacks.

| Student Weight (pounds) | Backpack Weight (pounds) |
|---|---|
| 105 | 15.5 |
| 81 | 15 |
| 97 | 30 |
| 133 | 13.5 |
| 78 | 10 |
| 120 | 16.5 |
| 100 | 18 |
| 116 | 16 |
| 98 | 13.5 |
| 110 | 15 |

| Student Weight (pounds) | Backpack Weight (pounds) |
|---|---|
| 127 | 17 |
| 112 | 19.5 |
| 118 | 17 |
| 140 | 21 |
| 108 | 15.5 |
| 110 | 16.5 |
| 132 | 19 |
| 105 | 14.5 |
| 83 | 16.5 |
| 96 | 13 |

According to a study by doctors at Johns Hopkins Medical Institute, students who carry more than the recommended weight can develop problems with their backs. Recommended weights are shown in the table below.

| If you weigh . . . | Your loaded backpack should weigh less than . . . |
|---|---|
| 70 pounds | 10.5 pounds |
| 80 pounds | 12 pounds |
| 90 pounds | 13.5 pounds |
| 100 pounds | 15 pounds |
| 110 pounds | 16.5 pounds |
| 120 pounds | 18 pounds |
| 130 pounds | 19.5 pounds |
| 140 pounds | 21 pounds |

*Data Presentation*

a. Create a visual summary using (student weight, backpack weight).

b. Calculate appropriate numerical summaries for measures of center and measures of spread for the weights of students' backpack data.

*Conclusion*

c. Write a paragraph describing your conclusions about the answer to the question "How heavy are the backpacks students carry?"

d. Write a paragraph describing your conclusions about the answer to the question "Do students carry backpacks that are too heavy for them?"

# Context and Units

1. Possible answers: Number of people in line at a fast-food restaurant, number of books you read last summer, weight of your backpack in pounds

2. Possible answers: Number of students; scores on standardized tests; student/teacher ratio; student/computer ratio; percent of teachers with graduate degrees; cost per pupil of educating students; percent of students who go to college after high school; attendance rates; suspension rates; dropout rates

3. 11,531,200,000 gallons

4. a. Total Gallons, South Atlantic

   b. Per-Capita Consumption, East South Central

   c. Retail Sales, East North Central

5. More gallons are consumed in the South Atlantic Region, but it has the greatest population. The per-capita column indicates gallons consumed per person.

6. a. Mean = 32.3 cents, median = 24.5 cents

   b. Divide the cost per serving by the serving size; 10.3¢, 12.5¢, 13.7¢, 13.7¢, 13.7¢, 14.3¢, 14.3¢, 15.7¢, 17.3¢, 18.8¢; the best buy is 12.5¢ per serving.

   c. \$2.16–\$8.28

   d. The range is 65 calories. Four of the bars have 45 calories. 45 calories is the mode and the median. Two brands have fewer than 45 calories. Four brands have more than 45 calories.

# Sorting and Counting

**1. a.** Possible categories: Blue, brown

**b.** Possible measurements: Time in minutes

**c.** Possible measurements: Length in inches, feet, or centimeters

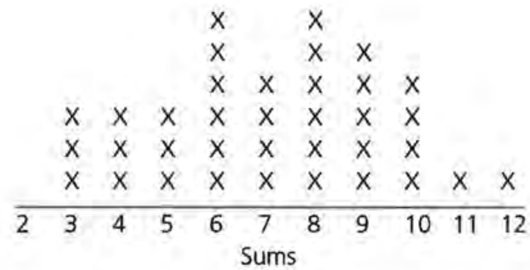
**2. a.** Possible answers: Mean = 17.5 years, median = 15 years, range = 32 years; three applicants have fewer than 11 years of experience, and five applicants have more than 11 years of experience.

**b.** Possible answer: There are 8 channels available.

**3. a.** Possible answers: Line plot or bar graph of one or more trials. A line plot for Trial 1 is shown in the next column. Students' work should indicate that the middle sums occur more frequently.

**b.** See table below.

**c.** Possible answer: The sums 5, 6, 7, and 8 occurred most frequently; 7 has the greatest frequency. Based on the data, if two dice were tossed 36 times, the number 7 would likely occur 5 or 6 times. The number 12 would likely occur 1 or 2 times.




**4.** The difference between the expected number of each sum and the number obtained is shown. Trials 1 and 3 were closest to the expected numbers. See table below.

**5. a.** 0 + 0.7 + 0.5 + 1.2 + 0 + 0.5 + 0.5 + 0.2 + 0.2 + 0.5 + 0.2 = 4.5

**b.** Yes

**c.** Possible answer: Less than 2, probably close to zero

**3. b.**

| Sum | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Mean** | 1 | 2.7 | 2.5 | 5.2 | 5 | 5.5 | 4.5 | 4.2 | 2.8 | 1.5 | 1.2 |
| **Median** | 0.5 | 3.5 | 2 | 5 | 4.5 | 5 | 5 | 4.5 | 3 | 1 | 1 |
| **Range** | 4 | 5 | 2 | 6 | 3 | 6 | 6 | 3 | 3 | 2 | 4 |

**4.**

**Difference Between Actual Number and Expected Number**

| Sum | Expected Number | Trial 1 | Trial 2 | Trial 3 | Trial 4 | Trial 5 | Trial 6 |
|---|---|---|---|---|---|---|---|
| 2 | 1 | 1 | 0 | 1 | 1 | 0 | 3 |
| 3 | 2 | 1 | 2 | 2 | 2 | 3 | 2 |
| 4 | 3 | 0 | 1 | 1 | 1 | 1 | 1 |
| 5 | 4 | 1 | 2 | 1 | 1 | 1 | 5 |
| 6 | 5 | 1 | 1 | 1 | 2 | 1 | 0 |
| 7 | 6 | 2 | 1 | 1 | 1 | 3 | 3 |
| 8 | 5 | 1 | 2 | 1 | 1 | 4 | 2 |
| 9 | 4 | 1 | 0 | 1 | 2 | 1 | 0 |
| 10 | 3 | 1 | 1 | 0 | 0 | 2 | 1 |
| 11 | 2 | 1 | 1 | 1 | 1 | 1 | 0 |
| 12 | 1 | 0 | 1 | 0 | 0 | 3 | 1 |
| **Closeness Scores** | | 10 | 12 | 10 | 12 | 20 | 18 |

# Using Percents

1. Students' grids could be colored so that each percent is represented by the appropriate number of squares. Or, students might construct a bar graph of the data.

2. a. 12 million tons

   b. 55.5 million tons

3. Shot charts will vary. Possible answer: One 3-point shot, four 2-point shots, and 6 missed shots

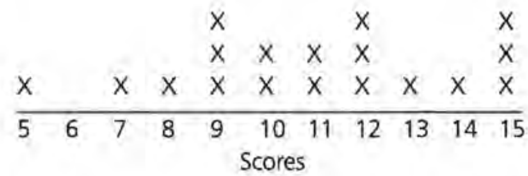
4. See table below.

5. (based on responses from Problem 4)
   a. About 48%

   b. About 52%

6. a. Possible answer: How well did the class perform on a quiz? How did a score of 11 compare to the rest of the scores?

   b. A line plot or box plot would be appropriate. A line plot is shown in the next column.




   c. Possible answer: The range of the scores is 10. There was a cluster of scores from 9 to 12. Ten of the 18 scores are in that range. Three students had perfect scores of 15. Eleven is the median score. There are eight scores above 11 and eight scores below 11. The mean score is 10.9. The class did not perform well on the quiz. Eight students scored below the mean. A score of 11 is just above the mean.

   d. A, 22%
   B, 22%
   C, 11%
   D, 28%
   F, 17%

| | **Landing Head Down** | | **Landing on Side** | | **Totals** |
|---|---|---|---|---|---|
| | **Number** | **Percent** | **Number** | **Percent** | |
| **Group 1** | 27 | 54% | 23 | 45% | 50 |
| **Group 2** | 10 | 30% | 23 | 70% | 33 |
| **Group 3** | 14 | 50% | 14 | 50% | 28 |
| **Group 4** | 34 | 52% | 31 | 48% | 65 |
| **Totals** | 85 | 48% | 91 | 52% | 176 |

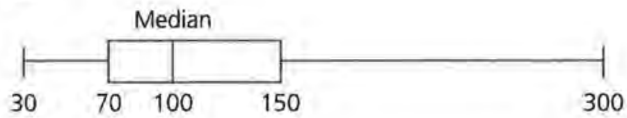
## UNIT IV QUIZ: SOLUTION KEY

# Measurement Data in Experiments

**1.** **a.** Median = 100 cm; mean = 118.2 cm

**b.** Median = 36 in.; mean = 36.9 in.

**2.** **a.**



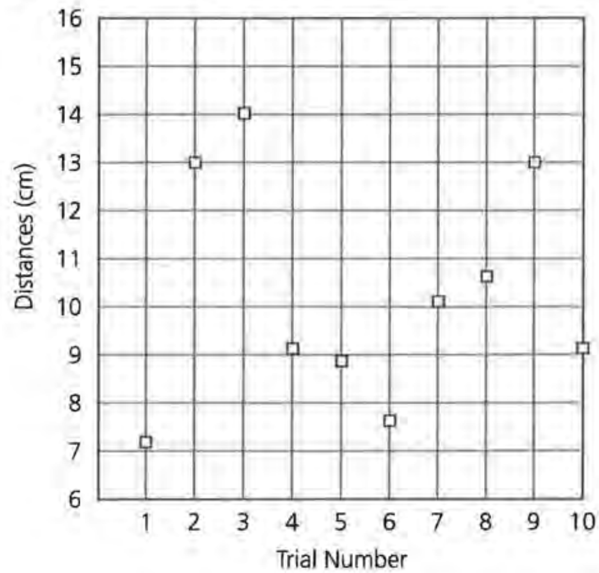

**b.**


**3.** **a.** Possible answer: The centimeter data were more spread out than the data in inches. There was a cluster of numbers around 100 cm, the length of a meter stick. There were gaps between 160 cm and 200 cm, between 200 cm and 275 cm, and between 275 cm and 300 cm. The range was 270 cm.

**b.** Possible answer: Most of the scores are clustered around 36 inches. 20% of the students chose 36 inches, the length of a yardstick. The range was 85 inches. However, disregarding an outlier of 100 inches, the range was 35 inches.

**4.** The estimates in inches were closer to the actual height. Perhaps students were more comfortable thinking about height in inches than centimeters because, in the United States, the height of a person is usually measured in inches.

**5.** **a.** A scatter plot or a bar graph with trial number on the horizontal axis and distance on the vertical axis and a box plot of the distances would be appropriate. A scatter plot is shown.

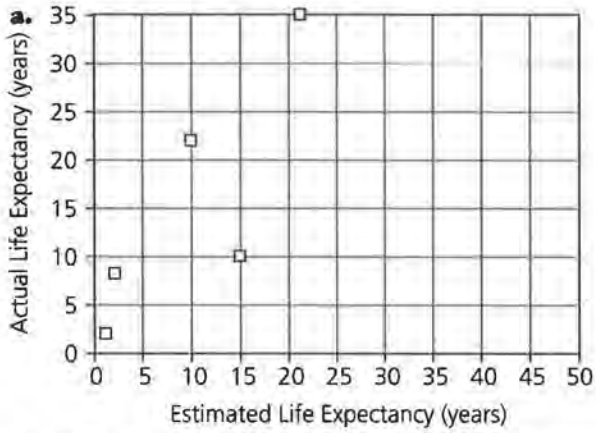



**b.** Mean = 10.3 cm; median = 9.6 cm

**c.** The range is 6.8 cm. There is a cluster around the center distance. The pattern of distances across the trials appears to be random.

# Mathematics in a World of Data

**1. a.**



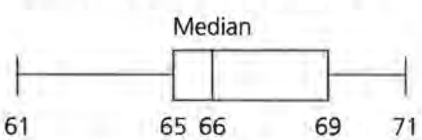

**b.** Draw the line $y = x$. Points above the line are underestimates. Points below the line are overestimates. Katie underestimated all but one of the life expectancies.

**2. a.** Katie, 38; Mandy, 31; Rachel, 25; Julia, 122; Jon, 29

**b.** Possible answer: The closeness scores on the table were calculated by adding the absolute values of the differences between estimates and actual life expectancies.

**c.** Julia

**3. a.** Data in the box plot shown have been converted from feet and inches to inches.




**b.** Mean = 66.5 inches; median = 66 inches

**c.** There is a cluster of heights from 64 to 67 in., and a smaller cluster from 69 to 71 in. There is a gap between 61 and 64 in. and a smaller gap between 67 and 69 in. The range of the data is 10 in.

**4. a.**

| | Cost (cents) | Percent of Cost |
|---|---|---|
| **Profit** | 56 | 17% |
| **Advertising** | 42 | 13% |
| **Coupons** | 82 | 25% |
| **Labor** | 19 | 6% |
| **Materials** | 82 | 25% |
| **Other** | 48 | 15% |

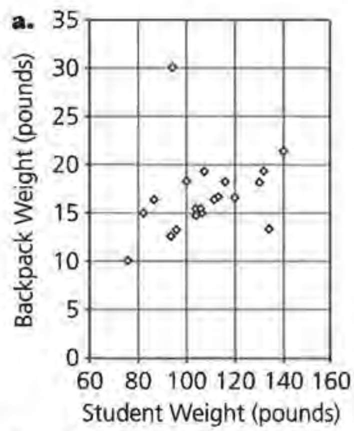
Be sure that students understand that due to rounding the total of the percent column is not 100%.

**b.** 25%

**c.** 38%

**d.** Grids could be colored so that each percent is represented by the appropriate number of squares. Or students might construct a bar graph of the data.

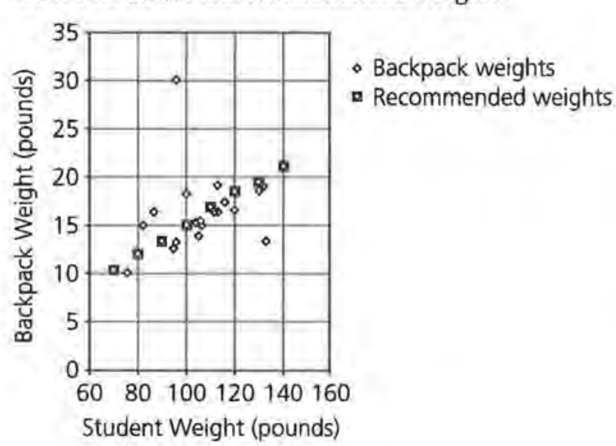
**5. a.**




**b.** Mean = 16.6 pounds; median = 16.25 pounds; range = 20 pounds

**c.** Possible answer: Based on the sample, most students carry backpacks that weigh 15 to 17 pounds. The range of weights is 20 pounds.

**d.** Possible answer: According to the table, about a fourth of the students carry backpacks that are too heavy for them. If a line were drawn connecting the recommended weights, backpacks above the line would be above the recommended weight.



ACTIVITY SHEET 1

# Lesson 3, Problem 1

NAME ______________________________

**Soda Consumption**

| Names | Day | Week | Month | Year |
|---|---|---|---|---|
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| **Averages** | | | | |

ACTIVITY SHEET 2

# Lesson 6, Problems 1 and 2

NAME ______________________________

**M&Ms Color Data**

| Color | Individual Estimate of Frequency | Actual Frequency | Class Mean Frequency | Difference (Actual Frequency Minus Class Mean Frequency) |
|---|---|---|---|---|
| ________ | ________ | ________ | ________ | ________ |
| ________ | ________ | ________ | ________ | ________ |
| ________ | ________ | ________ | ________ | ________ |
| ________ | ________ | ________ | ________ | ________ |
| ________ | ________ | ________ | ________ | ________ |
| ________ | ________ | ________ | ________ | ________ |
| Totals | ________ | ________ | ________ | ________ |

**Closeness score for actual counts** ________

**Closeness score for estimates** ________

## Project: What If Water Cost as Much as Cola?

NAME ______________________________

Multiply the number of gallons of water listed for each activity by a reasonable price for cola in your area.

Each time you use water, put a mark after the appropriate activity. Keep a tally of the times you use water. Then complete the chart and calculate how much money you would spend on water if it were as expensive as cola.

| Activity | Tally | Sum of Tallies | Water per Use (gallons) | Cost of Cola (per gallon) | Cost of Water per Use Using Cost of Cola | Total Cost |
|---|---|---|---|---|---|---|
| Washing dishes by hand | ______ | ______ | 10 | ______ | ______ | ______ |
| Washing dishes in an automatic dishwasher | ______ | ______ | 11 | ______ | ______ | ______ |
| Flushing toilet | ______ | ______ | 4 | ______ | ______ | ______ |
| Cooking and drinking | ______ | ______ | 3 per day | ______ | ______ | ______ |
| Washing hands | ______ | ______ | 1 | ______ | ______ | ______ |
| Brushing teeth (water running) | ______ | ______ | 2 | ______ | ______ | ______ |
| Shower | ______ | ______ | 18 | ______ | ______ | ______ |
| Taking a bath | ______ | ______ | 30 | ______ | ______ | ______ |
| Washing clothes | ______ | ______ | 30 | ______ | ______ | ______ |
| | | | | | Total | ______ |

# Project: What If Water Cost as Much as Cola?

NAME ______________________________

**Data Summary of Water Usage**

| Activity | Number of Gallons You Used in 5 Days | Average Number of Gallons Class or Group Used in 5 Days | Difference: *Amount You Used* Minus the *Average Amount Used* |
|---|---|---|---|
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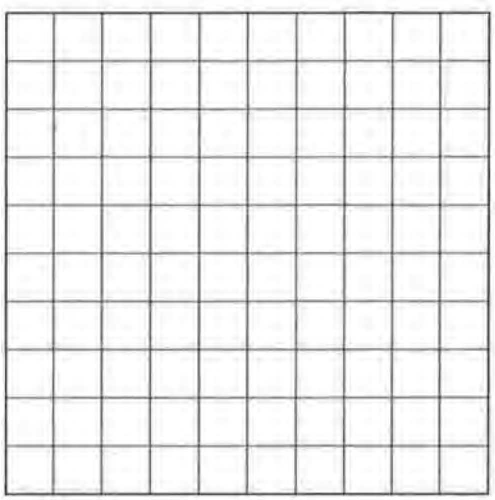
# Lesson 7, Problems 3 and 5

NAME ____________________________________________

**M&M Relative Frequency Color Data**

| | Individual Package Data | | | Class Data | |
|---|---|---|---|---|---|
| **Color** | **Estimated Frequency** | **Actual Frequency** | **Relative Frequency** | **Actual Frequency** | **Relative Frequency** |
| ______ | ______ | ______ | ______ | ______ | ______ |
| ______ | ______ | ______ | ______ | ______ | ______ |
| ______ | ______ | ______ | ______ | ______ | ______ |
| ______ | ______ | ______ | ______ | ______ | ______ |
| ______ | ______ | ______ | ______ | ______ | ______ |
| ______ | ______ | ______ | ______ | ______ | ______ |
| **Totals** | ______ | ______ | ______ | ______ | ______ |



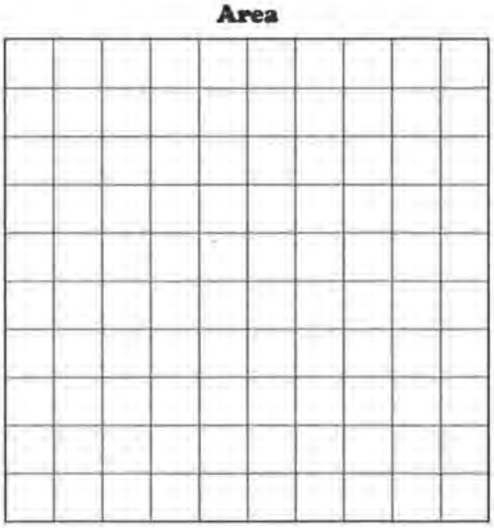
# Lesson 8, Problems 4–6 and 8

NAME ______________________________________________

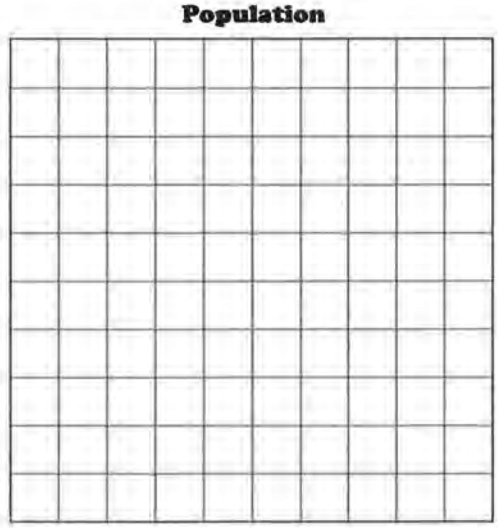
**Area and Population Data for the Earth**

| Continent | Area | | Population | |
|---|---|---|---|---|
| | 1000 sq. mi | Percent | 1000 Persons | Percent |
| North America | 9,400 | ______ | 277,000 | ______ |
| South America, Latin America, Caribbean | 6,900 | ______ | 450,000 | ______ |
| Europe | 3,800 | ______ | 499,000 | ______ |
| Asia | 17,400 | ______ | 3,286,000 | ______ |
| Africa | 11,700 | ______ | 795,000 | ______ |
| Australia | 3,300 | ______ | 26,000 | ______ |
| Antarctica | 5,400 | ______ | Uninhabited | ______ |
| Entire World | 57,900 | ______ | 5,333,000 | ______ |

**Area**



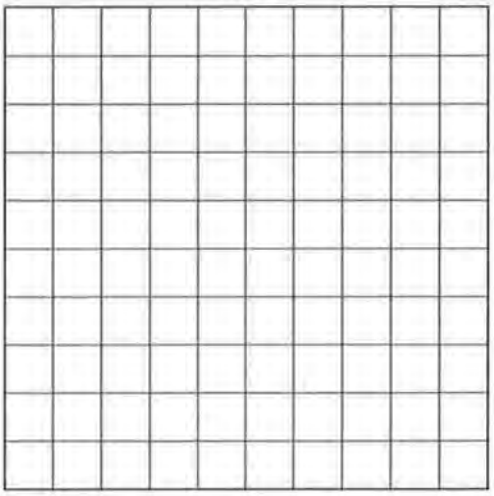
**Population**



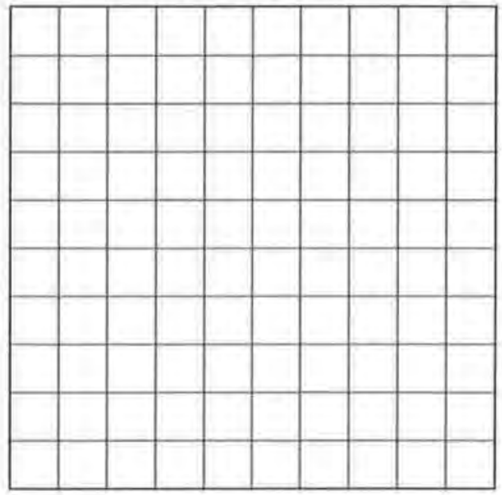
# Lesson 8, Problems 9 and 10

NAME ______________________________

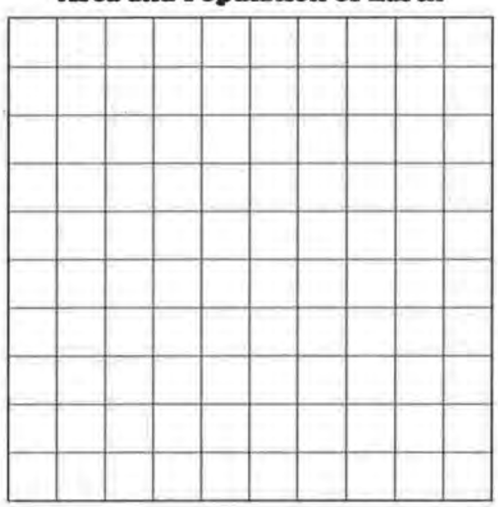
**Area**



**Population**



**Area and Population of Earth**



ACTIVITY SHEET 8

## Lesson 10, Problems 4–6 and 9–12

NAME ______________________________

### Game 1

| Player | Position | 2-Point Shots Attempted | 2-Point Shots Completed | 3-Point Shots Attempted | 3-Point Shots Completed | Percent Completed |
|---|---|---|---|---|---|---|
| Eric | Guard | ______ | ______ | ______ | ______ | ______ |
| Otto | Forward | ______ | ______ | ______ | ______ | ______ |
| Brian | Center | ______ | ______ | ______ | ______ | ______ |
| Jamie | Guard | ______ | ______ | ______ | ______ | ______ |

### Game 2

| Player | Position | 2-Point Shots Attempted | 2-Point Shots Completed | 3-Point Shots Attempted | 3-Point Shots Completed | Percent Completed |
|---|---|---|---|---|---|---|
| Eric | Guard | ______ | ______ | ______ | ______ | ______ |
| Otto | Forward | ______ | ______ | ______ | ______ | ______ |
| Brian | Center | ______ | ______ | ______ | ______ | ______ |
| Jamie | Guard | ______ | ______ | ______ | ______ | ______ |

### Game 3

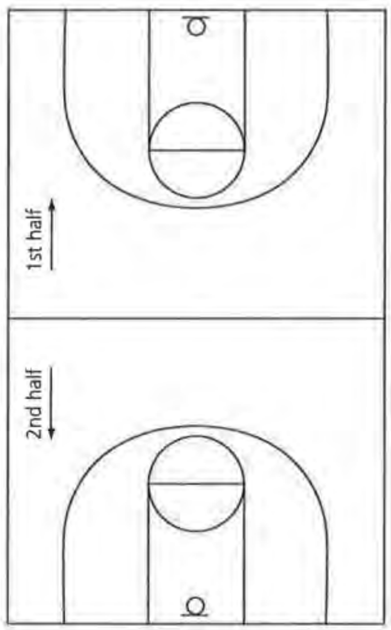
| Player | Position | 2-Point Shots Attempted | 2-Point Shots Completed | 3-Point Shots Attempted | 3-Point Shots Completed | Percent Completed |
|---|---|---|---|---|---|---|
| Eric | Guard | ______ | ______ | ______ | ______ | ______ |
| Otto | Forward | ______ | ______ | ______ | ______ | ______ |
| Brian | Center | ______ | ______ | ______ | ______ | ______ |
| Jamie | Guard | ______ | ______ | ______ | ______ | ______ |

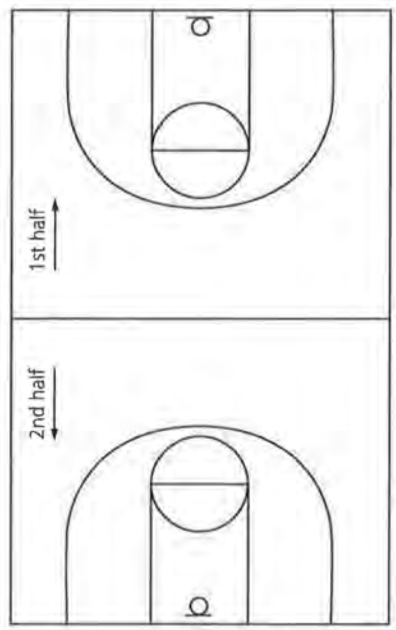
### Game 4

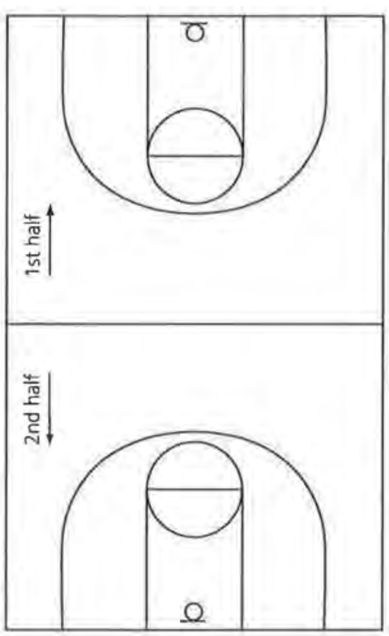
| Player | Position | 2-Point Shots Attempted | 2-Point Shots Completed | 3-Point Shots Attempted | 3-Point Shots Completed | Percent Completed |
|---|---|---|---|---|---|---|
| Eric | Guard | ______ | ______ | ______ | ______ | ______ |
| Otto | Forward | ______ | ______ | ______ | ______ | ______ |
| Brian | Center | ______ | ______ | ______ | ______ | ______ |
| Jamie | Guard | ______ | ______ | ______ | ______ | ______ |

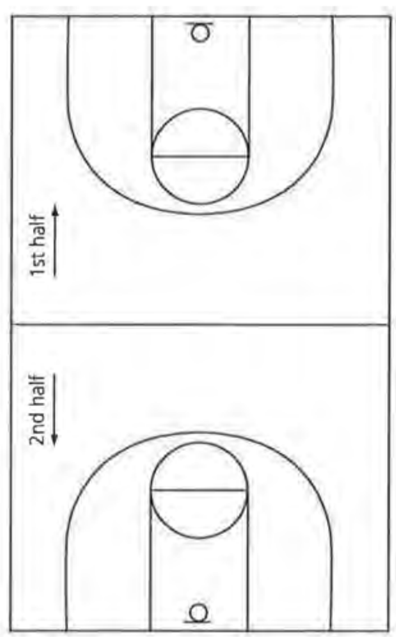
# Lesson 10, Problems 7 and 8

NAME ______________________________



# Lesson 10, Problems 13–16

NAME ______________________________

**Completion Percents**

| Player | Position | Percent Completed | | | |
|---|---|---|---|---|---|
| | | Game 1 | Game 2 | Game 3 | Game 4 |
| Eric | Guard | ______ | ______ | ______ | ______ |
| Otto | Forward | ______ | ______ | ______ | ______ |

**Eric's Shots**

| Eric | Average Distance from Basket | | Percent Completed | |
|---|---|---|---|---|
| | 2-Point Shots | 3-Point Shots | 2-Point Shots | 3-Point Shots |
| Game 1 | ______ | ______ | ______ | ______ |
| Game 2 | ______ | ______ | ______ | ______ |
| Game 3 | ______ | ______ | ______ | ______ |
| Game 4 | ______ | ______ | ______ | ______ |
| Averages | ______ | ______ | ______ | ______ |

**Otto's Shots**

| Otto | Average Distance from Basket | | Percent Completed | |
|---|---|---|---|---|
| | 2-Point Shots | 3-Point Shots | 2-Point Shots | 3-Point Shots |
| Game 1 | ______ | ______ | ______ | ______ |
| Game 2 | ______ | ______ | ______ | ______ |
| Game 3 | ______ | ______ | ______ | ______ |
| Game 4 | ______ | ______ | ______ | ______ |
| Averages | ______ | ______ | ______ | ______ |

# Lesson 11, Problems 2–4, 7, and 10

NAME ______________________________

**Paper-Cup Tosses**

| | Bottom | | Top | | Side | | Totals |
|---|---|---|---|---|---|---|---|
| | Number | Percent | Number | Percent | Number | Percent | |
| Group 1 | 7 | | 22 | | 171 | | 200 |
| Group 2 | 11 | | 22 | | 167 | | 200 |
| Group 3 | 12 | | 32 | | 306 | | 350 |
| Group 4 | 11 | | 25 | | 264 | | 300 |
| Group 5 | 10 | | 49 | | 241 | | 300 |
| Group 6 | 12 | | 26 | | 212 | | 250 |
| Group 7 | 2 | | 10 | | 34 | | 46 |
| Group 8 | 1 | | 27 | | 71 | | 99 |
| Totals | 66 | | 213 | | 1,466 | | 1,745 |

**Paper-Cup Tosses**

| | Bottom | | Top | | Side | | Totals |
|---|---|---|---|---|---|---|---|
| | Number | Percent | Number | Percent | Number | Percent | |
| Group 1 | | | | | | | |
| Group 2 | | | | | | | |
| Group 3 | | | | | | | |
| Group 4 | | | | | | | |
| Group 5 | | | | | | | |
| Group 6 | | | | | | | |
| Group 7 | | | | | | | |
| Group 8 | | | | | | | |
| Totals | | | | | | | |

**Lesson 12, Problem 5**

NAME ______________________

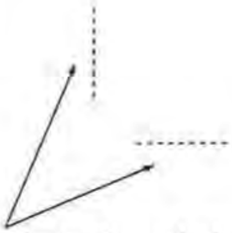
*Cut on the solid lines to produce the rear sides of the sleeve and the slide.*

Estimating Length

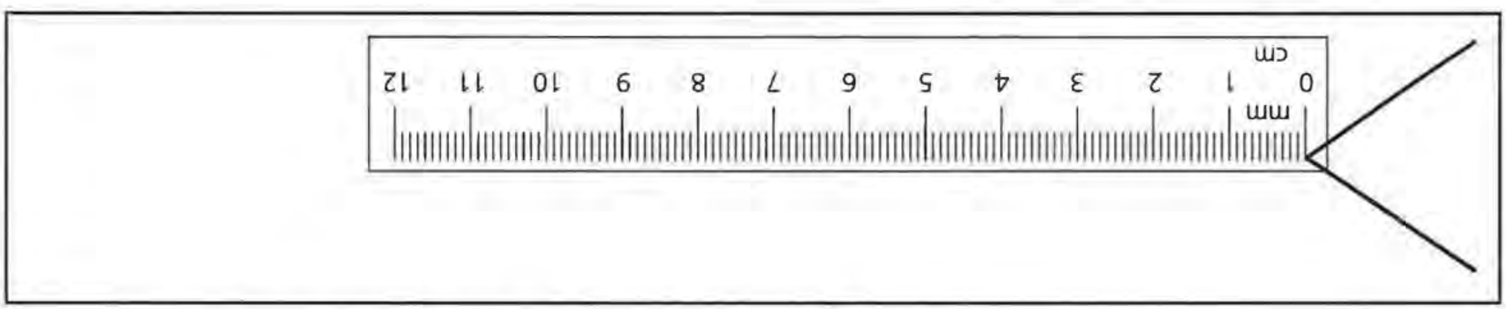
**Turn over to begin the experiment.**

Read the result here

Rear



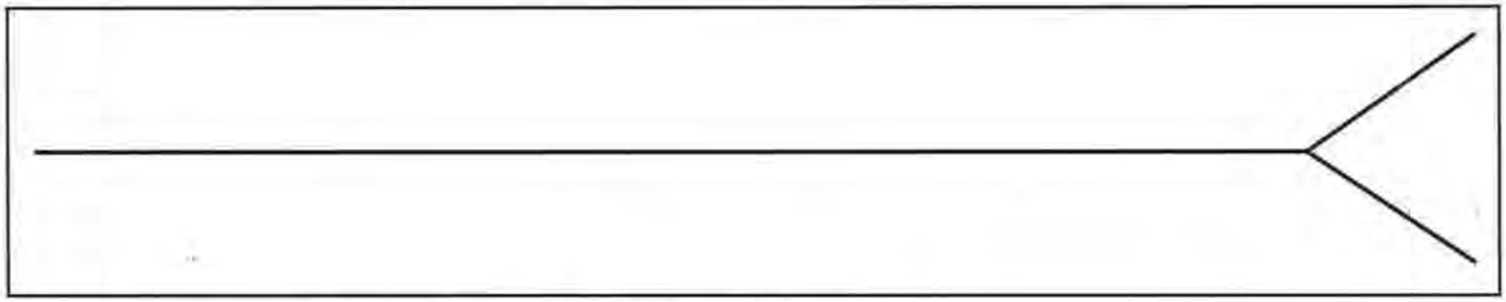
*Staples in the marked locations hold the front and back panels together and allow for the slide to move in the desired path.*



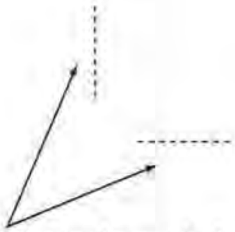

*Glue or tape these items to card stock, tagboard, or similar stiff material.*

## Lesson 12, Problem 5

NAME ____________



*Glue or tape these items to card stock, tagboard, or similar stiff material.*

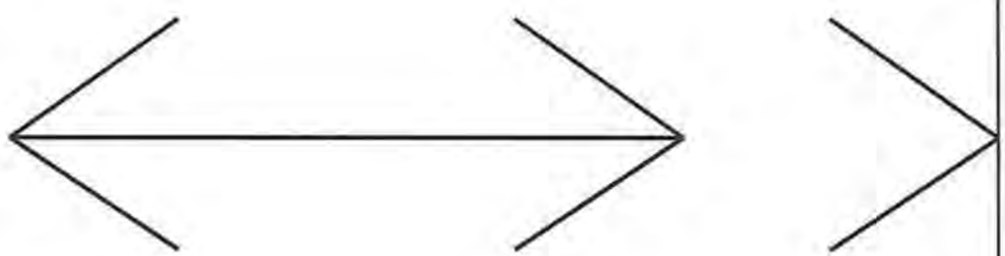


*Staples in the marked locations hold the front and back panels together and allow for the slide to move in the desired path.*

*Cut on the solid lines to produce the front sides of the sleeve and the slide.*

### Estimating Length

Front



Pull the slide out until you think the line segment on the right is as long as the line segment printed above.
Do not turn over until you have completed the task.

# Project: Waiting Time in the Lunch Line

NAME ____________________________________________

**Lunch-Line Waiting Time**

Date ______________________

Line ______________________

Start Time ______________________

End Time ______________________

**Lunch-Line Waiting Time**

Date ______________________

Line ______________________

Start Time ______________________

End Time ______________________

**Lunch-Line Waiting Time**

Date ______________________

Line ______________________

Start Time ______________________

End Time ______________________

**Lunch-Line Waiting Time**

Date ______________________

Line ______________________

Start Time ______________________

End Time ______________________

**Lunch-Line Waiting Time**

Date ______________________

Line ______________________

Start Time ______________________

End Time ______________________

**Lunch-Line Waiting Time**

Date ______________________

Line ______________________

Start Time ______________________

End Time ______________________

# Project: Waiting Time in the Lunch Line

NAME ____________________________________________

Data Recorder ______________________ Date __________

| Student | Line | Start Time | End Time | Total Time | Percent |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |
| 6 | | | | | |
| 7 | | | | | |
| 8 | | | | | |
| 9 | | | | | |
| 10 | | | | | |
| 11 | | | | | |
| 12 | | | | | |
| 13 | | | | | |
| 14 | | | | | |
| 15 | | | | | |
| 16 | | | | | |
| 17 | | | | | |
| 18 | | | | | |
| 19 | | | | | |
| 20 | | | | | |

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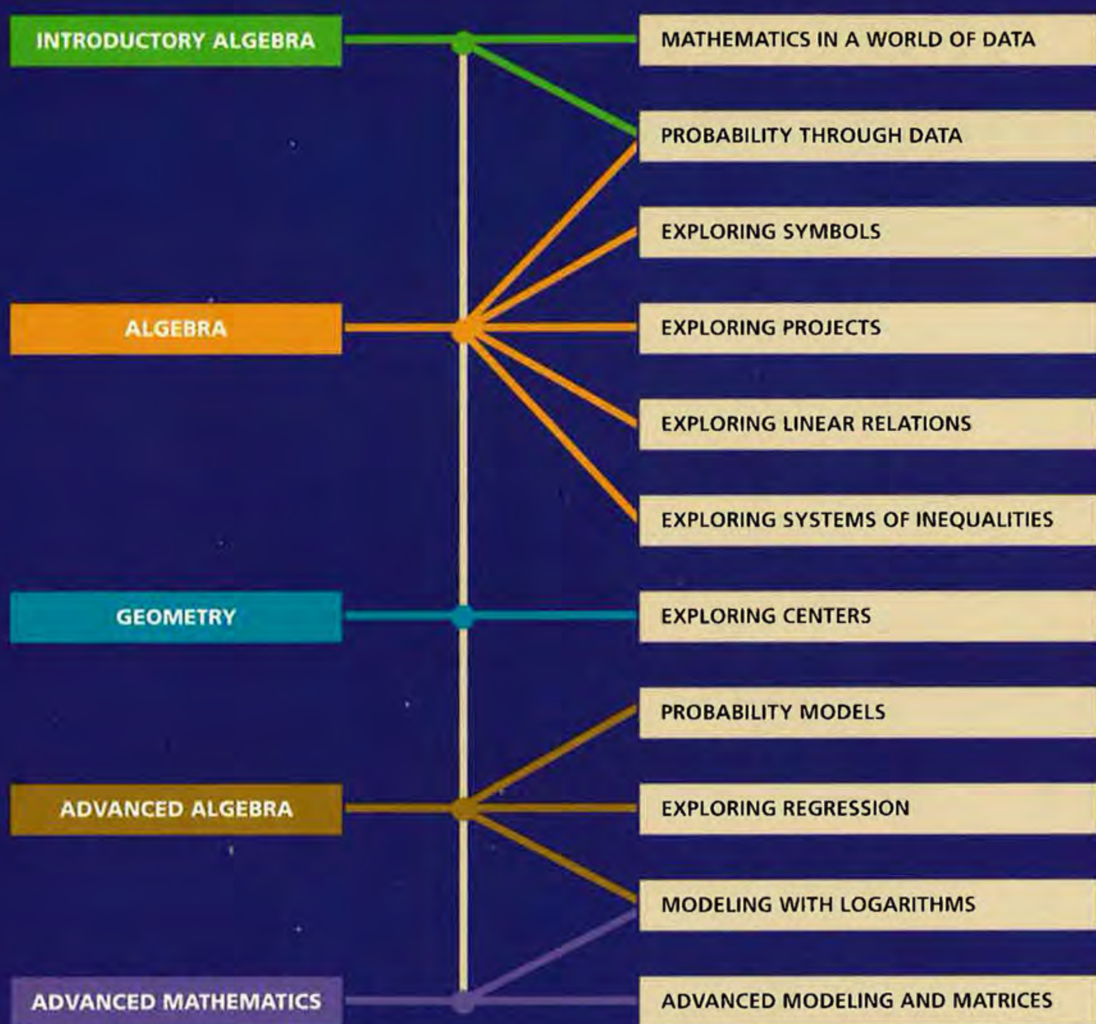


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